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THE TRANSFORMER

Magnetic Properties of Matter and Alternating Currents

THE TRANSFORMER

A Module on Magnetic Properties of Matter and Alternating Currents

SUNY at Binghamton

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The Transformer

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The Transformer

INTRODUCTION

You have previously studied direct-current (DC) circuits. In such circuits the charge always flows in the same direction because the terminals of the DC source (battery or power supply) always have the same sign. On the other hand, it is possible to have a voltage source whose terminals change sign (polarity) at regular intervals. This *alternating voltage* causes the direction of charge flow to alternate. A circuit in which the direction of current changes periodically is called an *alternating-current* (AC) circuit. In the United States, electric power is supplied at a frequency of 60 cycles per second, or 60 hertz (Hz).

A *transformer* is a device which is used primarily to increase or to decrease AC voltage. Although transformers have many applications, perhaps their most important use is in AC power transmission. Electric power is commonly transmitted over very long distances—perhaps hundreds of miles—from the generating station to the location at which it is used. Because the fraction of transmitted power which is wasted as heat in the transmission wire is lower for higher voltages, *step-up* transformers are used to increase the voltage at the generating station. At the other end of the transmission wire, *step-down* transformers in local substations reduce the voltage. The voltage is further reduced for home use by smaller step-down transformers located near each home. Inside the home or industry, other transformers are used to further change the voltage to meet the demands of electric motors, electronic circuits, and other electrical devices. Transformers are essential to the safe and efficient transfer of electric power.

PREREQUISITES

The following brief discussion is intended to refresh your memory of some essential concepts for understanding this module. These ideas are developed in greater detail in the Physics of Technology modules on *The Automobile Ignition System* and *The Solenoid*. Throughout this module we assume that you are familiar with DC circuits, as studied in *The Multimeter* module.

The mutual force of attraction or repulsion between two objects which do not touch each other (such as two magnets or two electric charges) is often explained in terms of a *field*. One object "alters" the space surrounding it, and the second object interacts with this alteration. Such an alteration of space itself is called a field. For example, a moving electric charge (or an electric current) alters the surrounding space in a manner which affects magnets brought into that space. This particular alteration of space is called a magnetic field, and it evidently interacts with magnets to exert forces on them.

We visualize this magnetic field as a collection of magnetic field lines, each of which is everywhere parallel to the magnetic field. Such lines can be plotted by using a small magnet (such as a compass needle) which always lines up with the magnetic field. The number of magnetic field lines crossing a given area, divided by that area, gives the strength of the magnetic field B in that region of space. In other words, the closer together the magnetic field lines are, the stronger the magnetic field. Figure 1 shows some magnetic field lines around a current-carrying wire.

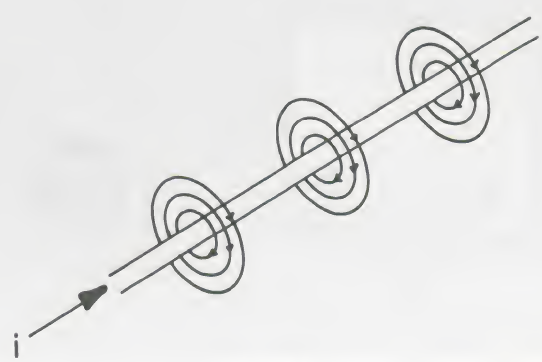

Figure 1.

Magnetic flux is a concept closely related to the concept of a magnetic field. Figure 2 shows short segments of magnetic field lines crossing a given region of space. If we imagine an arbitrary area A, perpendicular to the field direction, the magnetic flux Φ through the area A is equal to the number of magnetic field lines which cross A. In other words, if the magnetic field B is equal to the number of field lines per unit area, then the flux Φ can be calculated as

$$\Phi = B \cdot A$$

(for uniform field perpendicular to area A)

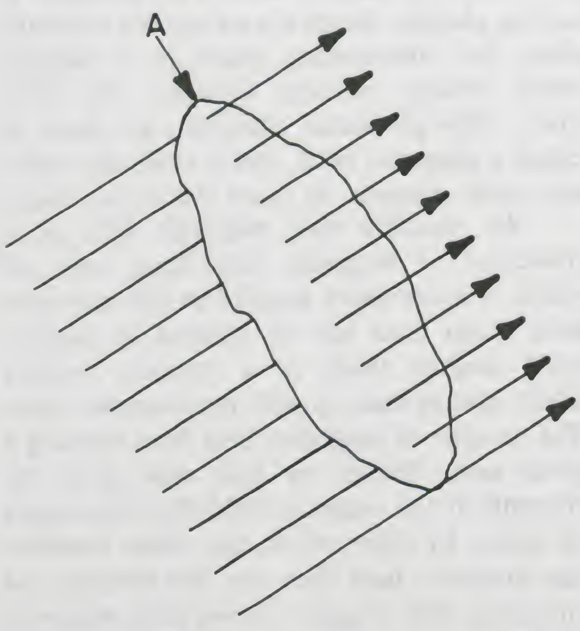

Figure 2.

The magnetic field B has units of teslas (T), and if area is specified in square meters, the flux Φ has units of webers (Wb).

In this module you will be especially concerned with flux through a loop of wire, or through a series of loops. Flux which passes through one loop of wire and then through a second is said to *link* the two loops (see Figure 3).

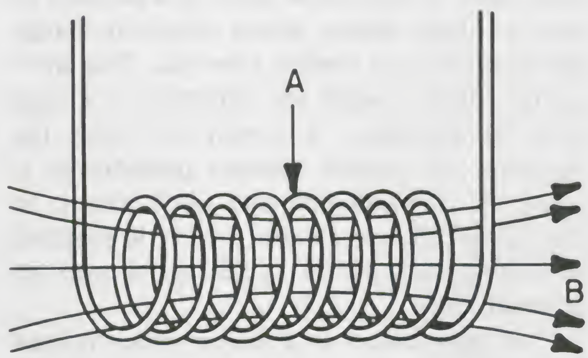

Figure 3. A flux which links three loops of wire.

Flux is an important concept because Faraday's law asserts that a changing magnetic flux induces an electromotive force, or voltage, in the region where the flux is changing. This *induced voltage* is proportional to the rate at which the flux changes. In this module you will study the effects of changing the flux through a series of wire loops. The induced voltage in the wire causes a current in the wire. This basic fact underlies all transformer operations.

SECTION A

TRANSFORMER CHARACTERISTICS

One simple kind of transformer is nothing but two coils of wire in separate circuits arranged physically so that as much as possible of the magnetic field created by one coil passes through the second coil. The two coils are usually wrapped around an iron core. Experiment A-1 will provide an opportunity for you to look at the parts of a transformer and to observe its basic operating characteristics.

EXPERIMENT A-1. Basic Transformer Characteristics

In this experiment you will study the parts of a simple experimental transformer and observe its operation under various conditions. You will use a *variable transformer* to provide a continuously variable AC-voltage source. In addition, you will use an *isolation transformer*–consisting of two identical coils–to isolate the circuit which you will study from the 120-V line voltage. It effectively removes the danger of being "shocked" by simultaneously touching a portion of a "hot" circuit and a ground. However, it does *not* eliminate the danger of shock when you touch two parts of the "hot" circuit at the same time.

CAUTION: Although the voltages used in experiments throughout the module are small, there is always some danger of severe electrical shock. **Always** turn the variable transformer to zero before connecting or disconnecting any part of the circuit.

Procedure

1. Connect the circuit shown in Figure 4.

   Turn the voltage of the variable transformer down to zero and plug it into a 120-V, 60-Hz wall outlet. For the first observation, use the *laminated* (layered) iron core. The coil connected to the isolation transformer (input coil) is called the *primary*; the coil connected to the AC voltmeter (output coil) is called the *secondary*. For the first observation, the primary coil should have about *twice as many* turns as the secondary. Note and record the number of turns of each coil.

2. Turn the variable transformer to its 10-V setting so that 10 V is across the primary of the transformer being studied. Read the voltmeter and record the voltage

Figure 4A.

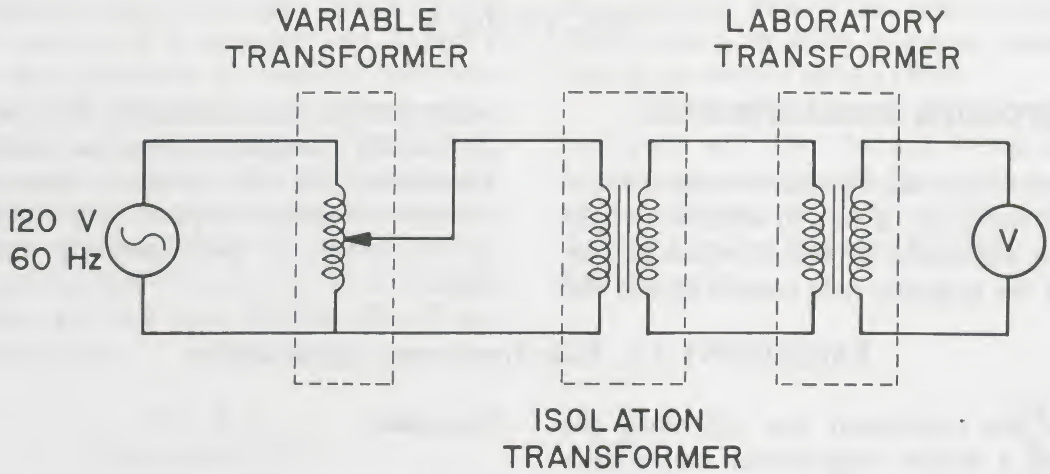

Figure 4B.

across the secondary. Turn the variable transformer voltage to zero. (**Always** do this before making any change in the circuits.)

3. Remove the primary coil (still connected to the isolation transformer) from the laminated core and place it on the table next to the core (see Figure 5). Reclamp the core and secondary without making other changes.

Figure 5.

Set the variable transformer to 10 V and record the voltage across the secondary.

4. Replace the primary on the laminated core. Remove the secondary, leaving it connected to the voltmeter, and place it on the table near the core (see Figure 6). Reclamp the core and primary without making any other changes.

Set the variable transformer to 10 V and record the secondary voltage.

Figure 6.

5. Remove the laminated iron core and replace it with the non-laminated iron core. Put both the primary and secondary on the core. Reclamp the core and coils as before.

Again set the variable transformer to 10 V and record the secondary voltage.

6. Replace the non-laminated iron core with the aluminum core, and repeat the measurement.

7. Replace the aluminum core with a wooden core and repeat the measurement.

8. Replace the wooden core with the laminated iron core. Replace the secondary coil with one that has more turns than the primary and reclamp the core and coils.

   Measure the secondary voltage when the primary voltage is 10 V.

9. Using no more than 10 V across the primary, see if there is any way you can arrange the coils so as to cause a voltage to appear across the secondary without using a solid core.

Questions

1. Does voltage ever appear across the secondary when either the primary or the secondary or both are not mounted on a core? If it does, describe the arrangement that produces the largest voltage across the secondary.

2. What cores produce the largest secondary voltages? Are there some cores for which no voltage could be detected across the secondary?

3. Is the voltage across the secondary larger when the number of turns in the secondary is larger than the number of turns in the primary, or when the number of turns in the secondary is smaller?

Discussion of Experiment A-1

The 60-Hz AC current through the primary coil creates a magnetic flux through the primary. The direction of the current, and thus the direction of the flux, changes each time the voltage "alternates." Some portion of this flux crosses the area surrounded by each of the loops of the secondary coil, inducing a voltage in each of the turns of the secondary, as determined by Faraday's law. The strength of this induced voltage depends on the rate at which the primary flux changes, and on the amount of primary flux which cuts through the secondary. Flux from the primary which also passes through the secondary is said to *link* the primary and the secondary. Thus the size of the induced voltage is related to the amount of flux linkage. (More accurately, the induced voltage depends on the rate of change of flux passing through the secondary.)

You probably observed that the largest induced voltage across the secondary occurs when both the secondary and primary are mounted on a laminated iron core. The secondary voltage decreased as the core was changed from laminated iron to iron, then to other materials.

Evidently the iron cores act to increase the flux linkage between primary and secondary. You will study core properties more carefully in Experiment A-3. For now, we simply state that the iron core must somehow guide magnetic field lines from the primary to the secondary.

AC-CIRCUIT COMPONENTS

What are the components of interest in a typical AC circuit? First, there must be a source of AC power. We will not discuss the generation of AC power. For our purposes, the combination of wall outlet, variable transformer, and isolation transformer will serve as a suitable AC source.

The component of prime importance in studying transformers is the *inductor.* Inductors are basically coils of wire having different sizes and shapes, depending on the uses for which they are intended. (See Figure 7.)

Figure 7.

Every inductor is characterized by a property called *inductance.* Inductance is a measure of the amount of magnetic flux that links the separate turns of wire in an inductor when current flows in the circuit. A single loop has an inductance of one henry (H) if one weber (Wb) of flux links the loop for each ampere (A) of current in the wire which forms the loop. It is easier to understand this *self-inductance* of a single inductor than the *mutual inductance* of two inductors. However, we will return to the latter before we are finished with our investigation of how a transformer operates.

Another important aspect of the behavior of an inductor is the relationship between the voltage across it and the current through it. This is easily studied by connecting an inductor to a readily available 60-Hz AC source and using an *oscilloscope* to display the time behavior of voltage across the inductor and other components in the circuit. You will make such a study in Experiment A-2. First you need to learn more about other AC-circuit components.

Another component often found in an AC circuit is a *capacitor.* A capacitor usually consists of two pieces of metal separated by a material which does not conduct electricity. Heavy-duty capacitors are often constructed by mounting two flat metal plates parallel to each other, but separated by a few millimeters. These plates are sometimes put in a can and immersed in oil to increase the ability of the capacitor to store energy. Small capacitors found in radio circuits may amount to nothing more than a pile of alternating layers of metal foil and waxed paper rolled up into the shape of a cylinder and covered with wax. (See Figure 8.)

Figure 8.

You may recall that, according to Coulomb's law, electric charges exert forces on one another. This means that electric charges which are held away from each other possess electrical potential energy relative to each other. The electrical potential energy per unit charge is called the electrical *potential difference* or, in more common terms, the *voltage.*

Thus, placing a positive charge on one plate of a capacitor and an equal amount of negative charge on the other side creates a voltage across the two sides. The amount of positive charge on one of the plates divided by the voltage across the two plates is a property of the capacitor called its *capacitance.* The larger the capacitance, the greater the charge on the plates for a particular voltage across the plates. A capacitor which holds a charge of one coulomb (C) with 1 V across it has a capacitance of one farad (F). Real capacitors are usually much smaller than this. We often measure capacitances in units of microfarads ($1\ \mu F = 10^{-6}$ F). A charged capacitor stores energy in the *electric field* that exists between the plates. Storing energy is an important function of a capacitor.

If a battery is connected across the capacitor, the charge on the capacitor increases until the voltage across the terminals of the capacitor is equal to the voltage of the battery. Until this equality is established, charge flows to the positive plate and away from the negative plate of the capacitor. (Actually, *negative* electrons flow to the negative plate, but it's often easier to think of positive charge as flowing.) No current flows from one side of the capacitor to the other through the insulator that separates them (see Figure 9). However, no serious error is made in solving circuit problems if one thinks of charge as flowing from the positive terminal of the battery, through the capacitor, and into the negative terminal of the battery. A capacitor blocks the flow of direct current (except while it is being charged up), but alternating current can pass back and forth through a capacitor in the sense described here.

The only other circuit component we need to be concerned about at this time is the resistor. You should already be familiar with the way resistors behave in DC circuits. They behave much the same in AC circuits, except that the current through the resistor and the voltage drop across it alternate at the same rate as the voltage input provided by the power supply.

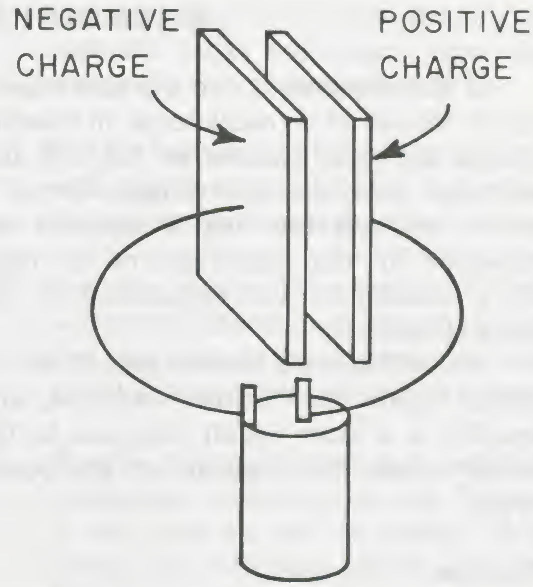

Figure 9. When a battery is first connected to a capacitor, a current is produced. When enough charge collects on the capacitor so that its voltage is equal to that on the battery, the current stops.

When all four elements—an alternating-voltage power supply, inductors, resistors, and capacitors—are connected into a circuit, some surprising things happen. Experiment A-2 will provide you with an opportunity to observe at least one of these for yourself.

The oscilloscope is a useful tool for studying properties of electrical circuit elements. An oscilloscope can be used to display a graph of voltage across an element versus time. In most applications, the scope is connected to the circuit so that the vertical direction on the oscilloscope screen measures voltage. When you look at the display screen you will see a pattern. Each different point of the pattern gives the voltage (across the circuit element you are studying) at a different time. As you look from left to right (on the display screen) you are seeing how the voltage changes from the starting time to some later time. The time scale is controlled by the dials on the oscilloscope. If you do not know how to use the oscilloscope, see the Appendix at the back of the module.

EXPERIMENT A-2. A Simple AC Circuit

In this experiment you will gain experience in the use of an oscilloscope to measure voltages and study waveforms. You will also determine how the AC voltages across a resistor, an inductor, and a capacitor are influenced by the magnitudes of the resistance, inductance, and capacitance of the circuit elements.

You will also see how the sum of the AC voltages across the resistor, inductor, and capacitor in a series circuit compares to the voltage across the terminals of the power supply.

Procedure

1. As before, connect an isolation transformer to the terminals of the variable transformer. Connect two resistors, two inductors, and two capacitors in series with the output terminals of the isolation transformer as shown in Figure 10. No two elements should be identical. Connect leads from the terminals of the isolation transformer to the *vertical deflection plate* terminals on the front of the oscilloscope.

2. Turn the voltage of the variable transformer to zero, and plug the transformer into a 120-V line. Turn the scope on and set the horizontal sweep time to about 2 ms/cm. Set the vertical scale to 5 V/cm.

3. Turn the voltage on the variable transformer to 10 V. You should see a sine-wave signal (Figure 11) on the scope face. Focus and center this trace and set its *amplitude* (Figure 12) so that it fills the scope face in the vertical direction. (If you need help, ask your instructor which knobs to turn.) Record, in scale divisions, the *peak-to-peak* distance this voltage covers on the scope. To calculate this voltage, multiply the number of scale divisions by the number of V/cm.

4. Turn the variable transformer to zero, and move the scope leads to the ends of one of the resistors.

   Turn the voltage back up to its original value, and record the peak-to-peak voltage across the resistor in scale divisions. You may have to reset the V/cm knob in order to measure this voltage.

5. Repeat the measurement for each resistor, each inductor, and each capacitor in turn. Be sure to turn the variable transformer to zero each time before touching the circuit.

6. Measure the voltage across the ends of the two resistors taken as a unit. (They must be next to each other in the circuit when you do this.)

7. Shut off the variable transformer and dismantle the circuit.

8. Connect a battery which supplies between 10 and 20 V across the vertical

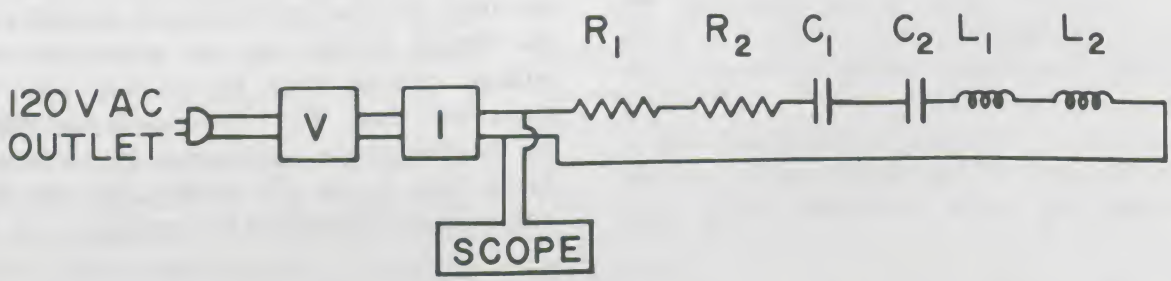

Figure 10. The series circuit is powered by a variable transformer (V) through an isolation transformer (I).

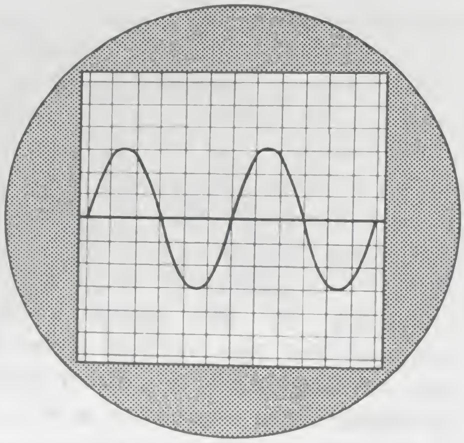

Figure 11.

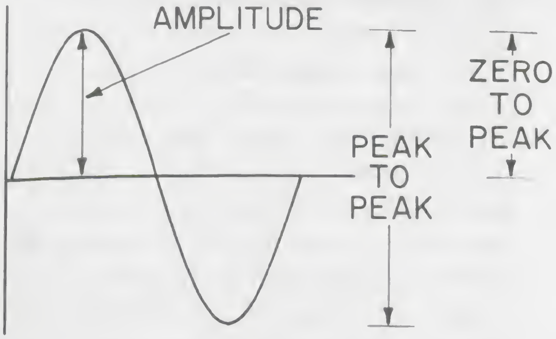

Figure 12.

deflection terminals of the scope. Set the scope to its DC mode. Record how many scale divisions the horizontal trace on the scope face jumps up or down when this connection is made or broken. Calculate the voltage of the battery.

Questions on Experiment A-2

1. The DC voltage measured corresponds to the voltage of the battery. What voltage does the peak-to-peak measurement in step 3 correspond to? Was it the same as the voltage indicated by the variable transformer?

2. Was the voltage across the resistor with the larger resistance larger than that across the smaller resistor?

3. Was the voltage drop across the inductor with the larger inductance larger than that across the smaller inductor?

4. Was the voltage drop across the capacitor with the larger capacitance larger than that across the smaller capacitor?

5. Was the voltage drop across the two-resistor combination equal to the sum of the two voltage drops across the individual resistors? Did you expect it to be?

6. Was the voltage drop across the isolation transformer (measured in step 3) equal to the sum of the six voltage drops across the individual circuit elements? Did you expect it to be?

Discussion of Preceding Questions

1. You probably noticed that the peak (zero-to-peak) voltage measured on the scope face is larger than the value indicated by the variable transformer (or measured by an AC voltmeter).

   An AC voltage has its peak value only at a single instant of time. As time passes, the voltage decreases to zero and then increases in the opposite direction to a peak value. What numerical value do we assign to such an alternating voltage? The choice is arbitrary. Customarily, the AC voltage and current are measured in terms of their heating effects on resistors. The effective (*rms**) value of an alternating current or voltage is equal to the direct current which produces the same heating effect on a given resistor. It turns out that this rms value is related to the peak value in a simple way.

$$V_{\text{rms}} = \frac{V_{\text{P}}}{\sqrt{2}} = 0.707\ V_{\text{P}}$$

*The term "root-mean-square" (rms) refers to the way in which the calculation of effective values is done. The procedure used takes the average value of the alternating quantity over a given time interval.

An rms voltage of 120 V corresponds to a peak voltage of 170 V. AC voltmeters and ammeters are calibrated to read rms values. However, the scope measures peak values, and these values must always be converted to rms values when they are to be compared with meter readings. Keep in mind that the peak value is one-half the peak-to-peak value. We shall leave off the subscripts indicating rms or peak values, except where they are needed for clarity.

2. For a given rms current, the rms voltage drop across a resistor is proportional to the resistance R. In Experiment A-2, you probably observed a larger voltage drop across the larger resistance.

3. In a simple DC circuit, consisting of a battery and resistors in series, the voltages across the resistors add as you go around the circuit. When all of the voltages are added together, the sum equals the voltage output of the battery. You might have expected the voltage across two adjacent resistors in an AC circuit to equal the sum of the two voltages across the individual resistors, and this should indeed be the case in practice.

4. For a given rms current, the rms voltage across an inductor is proportional to the inductance L. You should have observed a larger voltage drop across the larger inductance. This result is believable because a large inductance implies a lot of flux for a given current. Thus, there is a greater flux change as the current alternates, and a larger voltage is induced. (The rms voltage drop across an inductor is also proportional to the rate at which the voltage alternates, but you could not observe this since you have a constant 60-Hz source.)

5. For a given rms current, the rms voltage drop across a capacitor decreases as the capacitance C increases. You probably measured a smaller voltage drop across the larger capacitor. This result seems reasonable because the larger the capacitance, the smaller the voltage required to produce a given amount of charge on the plates. The current through the capacitor actually consists of a back-and-forth motion of charge around the rest of the circuit and between the two plates. This current was maintained at a constant rms value in Experiment A-2. (If the frequency of the applied voltage increases, the rms voltage across the capacitor decreases. For a direct current, the frequency of alternation is zero, and the capacitor totally blocks the current.)

6. Perhaps the most surprising observation in Experiment A-2 is that the sum of the rms voltages across the individual elements in a series AC circuit *is not* equal to the rms voltage output of the source of AC voltage. However, if we were able to measure the voltage drop across each element at the same instant in time, the sum *would* equal the voltage output of the source at that instant. Evidently, the various voltages do not alternate in such a way that they all reach their peak values simultaneously. We say that such voltages are not *in phase*. (See Figure 13A.)

   We know that charge does not "pile up" at any point in a circuit. This means that the *rate* at which charge is flowing in a series circuit—the current—is everywhere the same. Thus, we conclude that the currents in all elements of a series circuit are identical and always *in phase* with each other. Since we suspect that the voltages across various elements are not in phase, we also conclude that the voltage drops across some elements must be out of phase with the current through these elements. For some components, the voltage may reach its peak value earlier than the current (the voltage *leads* the current), while for others the current may reach its peak value before the voltage (the voltage *lags* the current). (See Figures 14 and 15.) We will investigate these effects more carefully in Section B of the module.

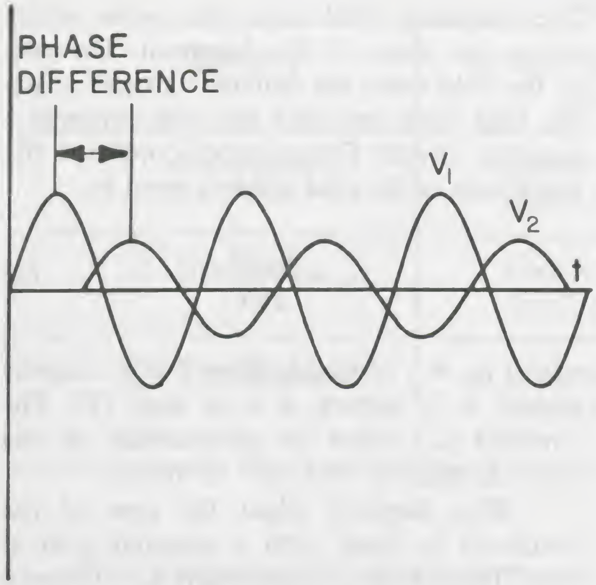

Figure 13A. Two voltages which are not in phase.

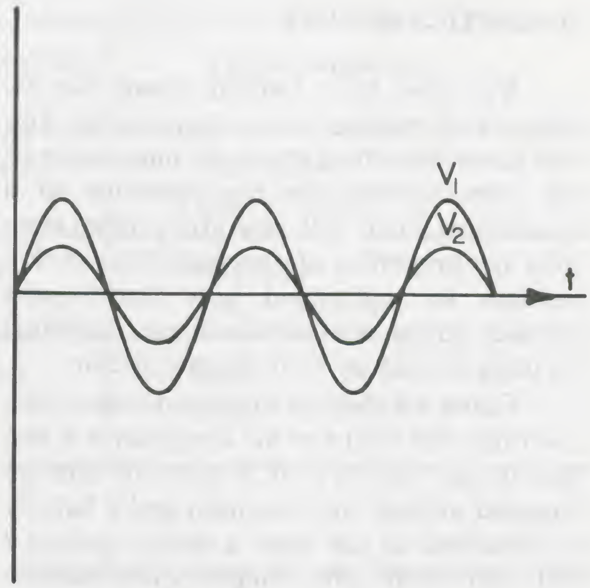

Figure 13B. Two voltages in phase. (Zero phase difference.)

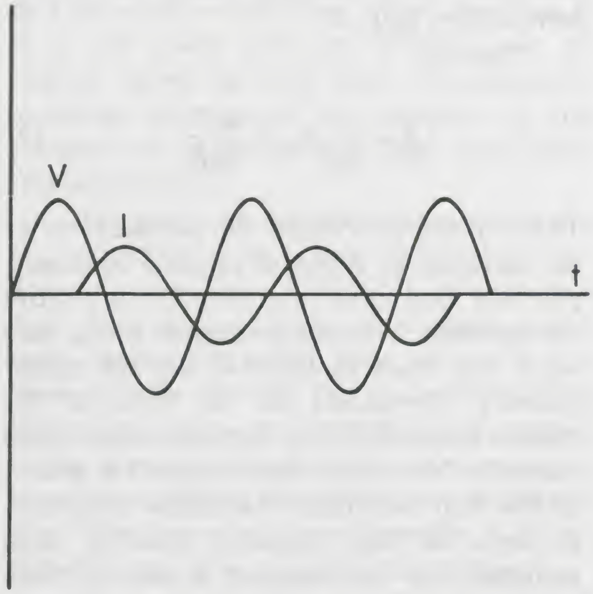

Figure 14. The voltage leads the current.

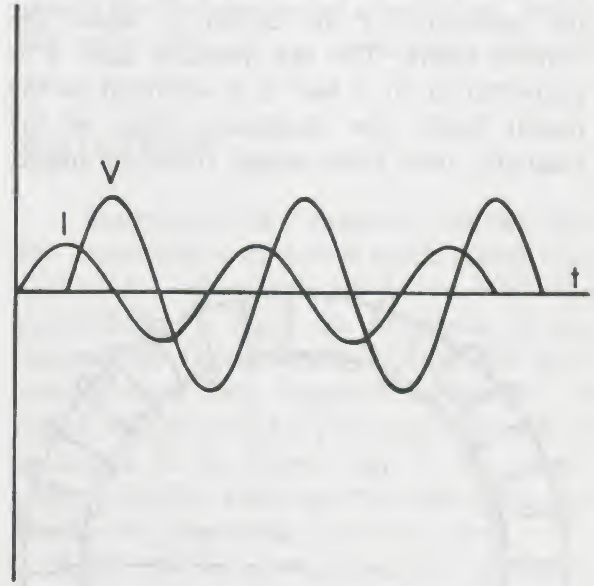

Figure 15. The voltage lags the current.

MAGNETIC CIRCUITS

You have been learning about the AC behavior of various circuit components. You also know something about the importance of the core material for the operation of a transformer. You will now study more carefully the properties of core materials and flux linkages to understand how transformers operate. These considerations are simplified by using an analogy to an electric circuit.

Figure 16 shows a doughnut-shaped hollow tube. The radius of the doughnut is R and that of the tube is r. If N turns of wire are wrapped around the doughnut and a battery is connected to the wire, a steady current I will exist in the wire. Ampere's law relates a magnetic field to the current which produces it. Its mathematical form is too complicated to write down in this module, but we can say that, according to Ampere's law, the magnetic field B is related to both the current I and to the geometry of the circuit in which the current exists. The law predicts that B is proportional to I and it is confined to the region inside the doughnut. None of the magnetic field lines escape from the inside.

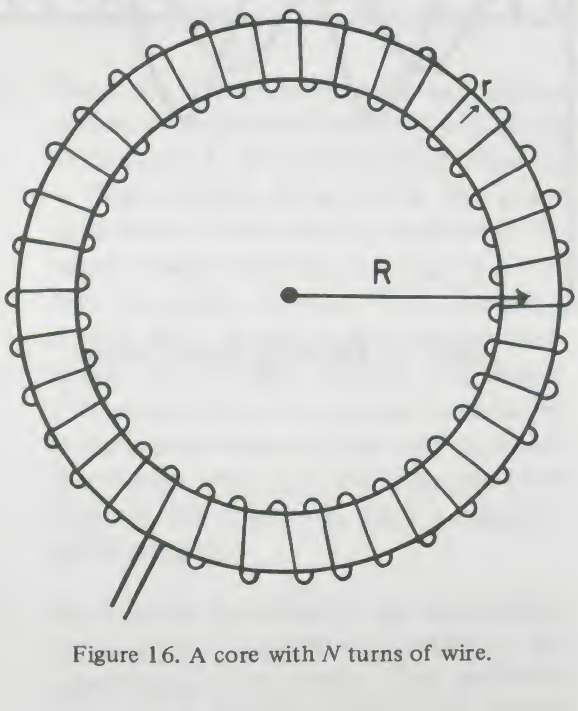

Figure 16. A core with N turns of wire.

The magnetic field lines are circles which follow the shape of the doughnut. The path of the field lines, the material through which the field lines pass, and the coils comprise a *magnetic circuit.* For an empty doughnut, the magnitude of the field inside is given by

$$B_0 = \frac{\mu_0 NI}{2\pi R} \qquad (1)$$

where μ_0 is a constant. When I is in amperes and R is in meters, B is in tesla (T). The constant μ_0, called the *permeability* of free space, is equal to $4\pi \times 10^{-7}$ (T·m/A).

What happens when the core of the doughnut is filled with a material such as iron? Think back to Experiment A-1. Evidently, the presence of the iron core increased the flux. It did this by increasing the magnetic field strength. It turns out that the field inside the core equals B_0, the field in empty space, multiplied by a constant, called the *relative permeability* (μ/μ_0).

That is:

$$B = \frac{\mu}{\mu_0} B_0 = \frac{\mu NI}{2\pi R} \qquad (2)$$

The quantity μ is called the *permeability* of the material. In tables of physical constants, you will find either permeability or relative permeability. The two are related by μ_0, and some care must be taken to use the proper quantity. Depending on the value of the relative permeability of the core material, the magnetic field inside the doughnut is greater or less than that inside the hollow doughnut. In terms of their magnetic behavior, most materials can be classified in one of three categories, as shown in Table I. The larger the relative permeability, the greater the magnetic field in the material for a given current I and number of turns N.

Only *ferromagnetic* materials such as iron, cobalt, nickel, and alloys of these metals respond significantly to an applied magnetic field. Iron is the most common ferromagnetic material and is almost always used for transformer cores.

Table I.

Category	Relative Permeability μ/μ_0	Typical Substance
1. Paramagnetic	slightly greater than 1.0	aluminum $\mu/\mu_0 = 1.0002$
2. Diamagnetic	slightly less than 1.0	beryllium $\mu/\mu_0 = 0.99989$
3. Ferromagnetic	much greater than 1.0	iron $\mu/\mu_0 = 50$ to 2000

What happens if we deviate from our ideal magnetic circuit? What if the current winding extends over only a fraction of the circumference of the doughnut? Or, what if the shape of the core is square or rectangular instead of round? If the core is iron, then the flux lines follow the shape of the core, even if it is not round and even if there are no current loops on one side. Ferromagnetic materials offer much less resistance to the presence of magnetic flux lines than other materials (or air).

The point of this discussion is that the iron transformer core is a magnetic circuit. The iron provides a "channel" for magnetic field lines. Because they are confined to the core, most of the flux lines set up in the primary pass through the secondary. Some slight *flux leakage* occurs at the sharp corners of the core. *Any* flux from the primary which does not intersect the secondary is leakage flux. One important consideration in transformer design is to minimize flux leakage.

The next question is, what happens if one cuts a gap in the iron core? Careful experiments have established that the lines representing the magnetic field are continuous; that is, they do not begin or end, but always form closed paths. Thus, if a small gap is cut in the doughnut-shaped iron core, the magnetic field should not be greatly different in the gap and in the iron. But, the flux might be different in two cores which are identical except for the presence of a gap in one. Is this the case? Let's do an experiment and find out.

RELUCTANCE

Experiment A-1 essentially shows that the permeability of iron is much higher than that of the other materials tested. When the permeability is high, the resistance of the material to the presence of magnetic field lines is low. This "magnetic resistance" is called *reluctance* and is given the symbol $\mathcal{R}$. It is related in an *inverse* way to the permeability. That is, the larger the reluctance, the smaller the permeability, and vice versa.

For the same experimental conditions [the same number of turns (N) and the same primary current (I)], a high-reluctance core contains a small flux, and a low-reluctance core contains greater flux.

EXPERIMENT A-3. **Measurement of Reluctance**

In principle, one can measure reluctance (and hence permeability) by applying a known current to the primary coil of a magnetic circuit and measuring the resulting flux. But how does one measure flux? One way is to use Faraday's law. That is, changing the flux in the primary induces a voltage in any other coil through which the flux passes. The method, then, is to wind a secondary coil around the core and measure the induced voltage in it when a known AC current is driven through the primary coil. The smaller the reluctance of the core material, the greater the flux in the core. The greater the flux, the greater the induced secondary voltage.

In this experiment, the primary circuit should consist of an AC-voltage source, an AC ammeter, and the primary coil on the iron core of the laboratory transformer. A convenient voltage source is the combination of an isolation transformer and a variable transformer, plugged into the 120-V line. The secondary circuit should consist of an AC voltmeter connected across the secondary coil on the core of the laboratory transformer. (See Figure 17.)

Procedure

1. Adjust the primary current to be about 1 A for each reading.

2. Read and record on the data page at the back of the module the voltage in the secondary when there is no gap in the core.

3. Reduce the variable transformer setting to zero, and place a paper *shim* between the two parts of the transformer core, as indicated in Figure 18. A 3-in X 5-in card makes a convenient shim. Make sure the clamps are tight. Be careful, the iron core gets hot!

4. Increase the voltage until the current returns to the same value used before and read the voltmeter in the secondary. Is there a difference between this reading and the previous reading?

5. Continue to add identical shims, and for each new gap thickness read the secondary voltage. Record the data in the table provided on the data page.

6. Plot $1/V_2$ on the vertical axis against the number of shims n on the horizontal axis.

Discussion of Experiment A-3

In the last part of the experiment, you have indirectly made a graph of the total

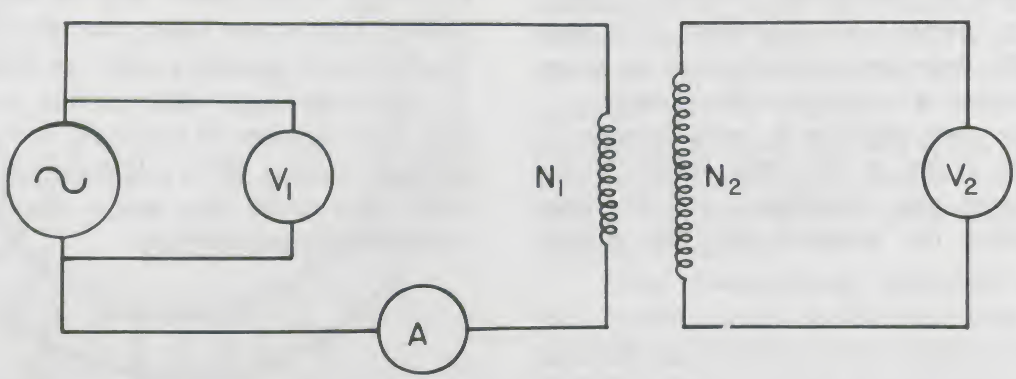

Figure 17. Circuit for Experiment A-3. (The electrical symbol for a transformer implies that both coils are on the same core.

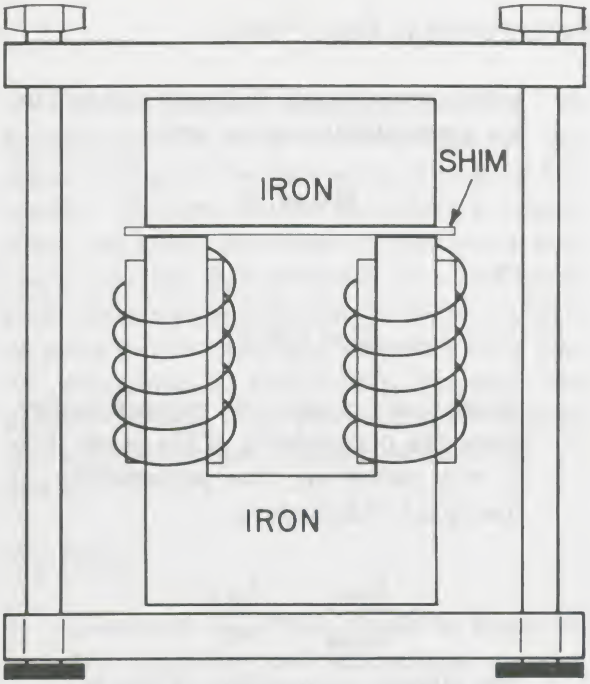

Figure 18. Paper shims inserted into the magnetic circuit.

reluctance $\mathcal{R}_T$ versus the width of the shimmed gap. This is because the secondary voltage V_2 is proportional to the flux through the secondary. Neglecting the small amount of flux leakage, that is just the flux in the magnetic circuit, and:

$$V_2 \propto \Phi$$

But the less the total reluctance, the greater the flux:

$$\mathcal{R}_T \propto \frac{1}{\Phi}$$

Thus:

$$\mathcal{R}_T \propto \frac{1}{V_2}$$

The permeability of iron is very large and thus its reluctance is extremely small. Therefore, if one were to add a thin shim of iron to increase the path length of the original magnetic circuit, the total reluctance of the circuit would be increased only slightly. On the other hand, the reluctance of paper is very much greater, close to that of empty space. Thus, inserting a paper shim into a magnetic circuit can add quite a bit to the total reluctance.

The fact that adding a material of high reluctance in "series" in a magnetic circuit with iron greatly increases the total of the circuit suggests an analogy to DC circuits. Adding a high *resistance* in series with a very low resistance produces a high total resistance.

$$R_T = R_1 + R_2$$

If R_2 is very large, then R_T is also very large. This suggests that we might write the total reluctance of a magnetic circuit in a similar way.

$$\mathcal{R}_T = \mathcal{R}_{iron} + \mathcal{R}_{paper} \tag{3}$$

If $\mathcal{R}_{paper}$ is very large, then the total reluctance is also very large.

For Experiment A-3, $\mathcal{R}_{paper}$ was varied in a regular way:

$$\mathcal{R}_{paper} = n\mathcal{R}_s$$

where n is the number of shims and $\mathcal{R}_s$ is the reluctance of one shim.

In this case, Equation (3) can be rewritten as:

$$\mathcal{R}_T = \mathcal{R}_{iron} + n\mathcal{R}_s \tag{4}$$

The quantities $\mathcal{R}_{iron}$ and $\mathcal{R}_s$ are constants. Thus Equation (4) predicts that a graph of total reluctance $\mathcal{R}_T$ versus n should be a straight line. Since $\mathcal{R}_T$ is proportional to $1/V_2$, your graph of $1/V_2$ versus n is, in fact, a graph of $\mathcal{R}_T$ versus n.

Questions

1. Is your plot of $1/V_2$ versus n a straight line?

2. What is the meaning of the value of $\mathcal{R}_T$ when $n = 0$?

3. From your graph, for what value of n is the value of $\mathcal{R}_T$ twice the value corresponding to $n = 0$?

4. Measure the thickness of one card shim (ℓ_s) in millimeters. To what thickness shim (in mm) does your answer to Question 3 correspond? This gives you a measurement of one half of the gap ($n\ell_s = \frac{1}{2}\ell_{gap}$). Why half?

5. Reluctance is a measure of the difficulty of producing a given magnetic flux in a material. It seems logical that the longer the magnetic circuit of a given material, the more difficult it is to maintain a flux in the circuit. Careful experiments show that, indeed, the reluctance of a piece of material is proportional to its length:

$$\mathcal{R} \propto \ell$$

When the reluctance of the core plus the shims is twice the value of $\mathcal{R}_{iron}$ for a circuit with no gap, as in Question 3:

$$\mathcal{R}_T = 2\,\mathcal{R}_{iron} = \mathcal{R}_{iron} + n\mathcal{R}_s$$

Thus:

$$\mathcal{R}_{iron} = n\mathcal{R}_s$$

If $\mathcal{R}$ is proportional to length and to $1/\mu$, the permeability, we can write

$$\mathcal{R} \propto (\ell/\mu)$$

or

$$\mathcal{R} = k\,(\ell/\mu)$$

where k is a constant of proportionality. Since the permeability of the paper shim is very nearly μ_0, the permeability of free space, this leads to:

$$k\frac{\ell_{iron}}{\mu_{iron}} = k\frac{\ell_{gap}}{\mu_0}$$

The relative permeability of iron is then:

$$\frac{\mu_{iron}}{\mu_0} = \frac{\ell_{iron}}{\ell_{gap}}$$

Calculate the relative permeability of the iron core. Measure ℓ_{iron} in the center of the core rather than either the inside edge or the outside edge. (If your plot of $1/V_2$ versus n deviates from a straight line, your value of the relative permeability will be only an approximation.)

EXPERIMENT A-4. Turns Ratio

Let's consider another question. In Experiment A-1, you observed that the induced voltage in the secondary is larger if the number of turns in the secondary is larger. From the initial discussion of reluctance, you know that the flux produced by a coil (such as the primary) is proportional to the number of turns in the coil (from Ampere's law). Can we determine a relationship between the primary voltage V_1, the secondary voltage V_2, and the number of turns of each coil N_1 and N_2?

Procedure

1. Construct the circuit shown in Figure 4. Use a primary coil with a large number (N_1) of turns and a secondary coil with about half as many turns ($N_2 \approx \frac{1}{2} N_1$). Record N_1 and N_2 on the data page.

2. Set the primary voltage V_1 to 3 V and record the secondary voltage V_2 in the table on the data page. Increase the primary voltage, 3 V at a time, until you reach 30 V, or the maximum safe voltage for the coil you are using. Record the secondary voltage for each primary voltage.

3. Plot V_1 versus V_2. Is V_1 proportional to V_2? (V_1 is proportional to V_2 if the graph of V_1 versus V_2 is a straight line passing through the origin.)

4. Turn the variable transformer to zero and connect a different secondary coil. Record the number of turns N_2. Measure and record the secondary voltage V_2 when V_1 is 15 V. Repeat for four different secondary coils, recording N_2 and V_2 for each coil.

5. Plot V_2 versus N_2. Is V_2 proportional to N_2? What can you say about the ratio V_2/N_2 for the different secondary coils? How does V_2/N_2 compare to V_1/N_1?

6. Turn the variable transformer to zero and connect a different primary coil, along with the secondary coil used in step 1. Record the numbers of turns N_1 and N_2. Set the primary voltage to 15 V and record the secondary voltage V_2. Repeat for four different primary coils.

7. Plot $1/N_1$ versus V_2. Are the two quantities proportional?

You probably concluded that the secondary voltage V_2 *is* proportional to the primary voltage V_1. If you were to compute the *slope* of the graph of V_1 versus V_2, you should find that it is equal to the *ratio* of the numbers of turns in the primary and secondary. You also should have seen that the secondary voltage V_2 *is* proportional to the number of turns N_2 on the secondary coil, and that the secondary voltage V_2 *is* proportional to the inverse ($1/N_1$) of the number of turns on the primary. Mathematically,

$$V_2 \propto V_1 \quad \text{(rms values)}$$

$$V_2 \propto N_2$$

$$V_2 \propto (1/N_1)$$

We can combine these into a single proportionality relation

$$V_2 \propto \frac{V_1 N_2}{N_1}$$

This proportionality can be written as an equation by using a proportionality constant k,

$$V_2 = k\,\frac{V_1 N_2}{N_1}$$

For an *ideal* transformer, the value of k is 1 and

$$V_2 = \frac{V_1 N_2}{N_1}$$

or,

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \tag{5}$$

Equation (5), called the *transformer equation,* can be derived from Ampere's law and Faraday's law. It says that the voltage ratio produced by a transformer is equal to the turns ratio of the two coils.

Example 1. The electric power lines which distribute electricity to neighborhoods operate at 12,500 V (rms). Transformers, usually mounted on poles, step this voltage down to 240 V for household use. What is the ratio of primary turns (N_1) to secondary turns (N_2)?

Solution. The transformer equation can be written as

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{12{,}500\text{ V}}{240\text{ V}} = 52$$

There should be 52 turns in the primary coil for every turn in the secondary coil.

Although we have derived the transformer equation for just one frequency (60 Hz), the fact that the equation depends only on the flux linkage shows that it should also hold for all other frequencies as well. In actual operation other factors, which will be discussed later, may cause a transformer to deviate from the equation if the frequency changes.

Problem 1. The turns ratio of a high-voltage x-ray transformer is 800 to 1. If 120 V are across the primary, what will be the secondary voltage?

Problem 2. What turns ratio is required for a step-down transformer to provide 12.6 V from a 120-V input? If the primary coil has 200 turns, how many turns must the secondary winding have?

Question 1.

a. Would it be bad for a power transformer to deviate from the transformer equation as the frequency of the input voltage is changed? Why?

b. Answer the same question for the audio-output transformer of a high-fidelity amplifier.

SUMMARY

A transformer is a device that is capable of increasing or decreasing an AC voltage. An efficient transformer requires good flux linkage between its two coils. This linkage is best accomplished by using a magnetic circuit made of laminated iron.

An inductor is a coil of wire designed so that magnetic flux produced by current in the coil links the various turns of the coil. This causes a voltage across the coil whenever the flux changes, as described in Faraday's law.

A capacitor is usually made of two pieces of metal close enough together to effectively store electric charge and energy when a voltage difference is created between them.

The rms voltage is related to the peak voltage by the expression:

$$V_{\mathrm{rms}} = \frac{1}{\sqrt{2}} V_{\mathrm{P}}$$

In a series AC circuit, a larger rms voltage is measured across a larger resistance than a smaller one, across a larger inductance than a smaller one, and across a smaller capacitance than a larger one. The sum of the rms voltages across all the components in a series AC circuit is not necessarily equal to the rms voltage across the source of AC power.

The magnetic flux through an ideal, doughnut-shaped core wrapped with wire is confined to the core and, among other things, it depends on the core material, as follows:

a. Compared to the flux with an empty core, the flux is slightly increased if the core is a paramagnetic material.

b. The flux is slightly decreased from the empty-core case if the core is a diamagnetic material.

c. The flux is greatly increased if the core is a ferromagnetic material.

Magnetic circuits with ferromagnetic cores, which differ from the ideal, doughnut-shaped core with wire turns all the way around, still confine most of the flux to the core.

The product (NI) of the current and the number of turns in the coil is proportional to the magnetic flux produced.

The "resistance" of a substance to the presence of a magnetic field is called reluctance. The larger the reluctance, the smaller the flux through the core. The reluctance is inversely proportional to the permeability of the core material.

Flux lines are continuous. Furthermore, they will not diverge much as they pass across a short gap in a ferromagnetic circuit.

If a gap is cut in a magnetic circuit, the total reluctance $\mathcal{R}$ can be found by adding the reluctance of the gap to the reluctance of the rest of the material in the circuit. The flux through a ferromagnetic circuit is greatly reduced when a small air gap is cut because the reluctance of the gap is large compared to that of iron. The relative permeability of iron is much larger than that of air.

The voltage across the secondary coil is larger than that across the primary coil if the number of turns in the secondary coil is larger than the number in the primary coil. The secondary voltage of a transformer is proportional to the primary voltage and to the number of turns in the secondary coil. The secondary voltage is inversely proportional to the number of turns in the primary coil. In an ideal transformer,

$$V_2 = \frac{V_1 N_2}{N_1}$$

SECTION B

CAPACITORS

In Experiment A-2, you observed some properties of capacitors as AC-circuit elements. In some respects, they behave like resistors. If an AC-voltage source is connected directly across a resistor, the voltage drop across the resistor V_R is proportional to the current I through it. The constant of proportionality is the resistance R and the relationship among V_R, I, and R is known as Ohm's law:

$$V_R = IR \tag{6}$$

If the current alternates, then the voltage drop V_R alternates in the same way. These two quantities reach their peak values simultaneously; that is, they are always *in phase.* Since Ohm's law holds at all times, it holds for the peak values of V_R and I. The rms values for both V_R and I are $\sqrt{2}$ times smaller than the corresponding peak values, so Ohm's law holds for rms values also.

In what ways do capacitors behave as resistors, and in what ways do they behave differently? Even though charge does not flow through a capacitor, AC current does exist in a series circuit containing capacitors.

In Experiment A-2, when you looked on the scope at the voltage drop across a capacitor, you saw a sine-wave pattern. The observed pattern for the voltage drop across a resistor in series with the capacitor was also a sine wave. We have already argued that there is no phase difference in the current at various points in the AC series circuit. Since voltage and current through a resistor are in phase, and since voltage across a resistor is proportional in size to the current through a resistor, a graph (scope trace) of voltage across a resistor versus time is also a graph of current versus time. But since the current everywhere in the circuit has the same phase, the waveform of the voltage across the resistor gives the current waveform anywhere in the circuit. The only difference is a constant "scale factor" R (the resistance). So in the following experiments, keep in mind the fact that a measurement of voltage across a resistor is essentially a measurement of current in the resistor.

What we still don't know is this:

1. Is the (instantaneous) voltage across a capacitor at all times proportional to the (instantaneous) current through it? That is, can we find an equation like Ohm's law that applies to a capacitor at every instant of time?

2. Does the rms voltage across a capacitor increase in proportion to the rms current through it? Can we discover an equation like Ohm's law that applies to rms capacitor currents and voltages? If so, what determines the constant of proportionality?

You can answer these questions by doing Experiment B-1 carefully. We will seek answers to the second question in Part A and to the first question in Part B.

PART A. Capacitive Reactance

In this part of the experiment you will try to determine if there is a constant X_C for a capacitor such that

$$V_C = IX_C \qquad (7)$$

You will measure peak-to-peak voltages, but keep in mind that the rms values are proportional to the peak values.

Procedure

1. Connect a series circuit as shown in Figure 19. As before, use the variable transformer and an isolation transformer as your source of AC voltage. Be sure the voltage setting of the variable transformer is initially zero.

   The capacitor should have a capacitance of approximately 1 μF (microfarad). You should have another capacitor available whose capacitance is two or three times larger. Any resistor with a resistance less than 100 Ω will do. The purpose of the resistor is to allow current measurement with the oscilloscope. Its resistance is so small that, as you will see, it does not otherwise affect the circuit.

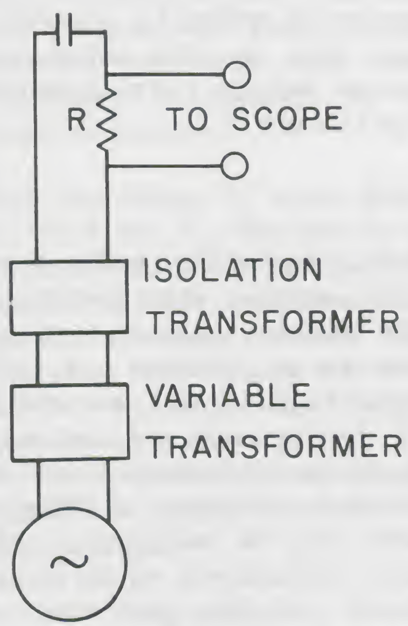

Figure 19. Circuit for Experiment B-1.

2. Connect the vertical deflection terminals of the oscilloscope to the ends of the resistor. The amplitude of the sine-wave pattern you will see on the scope face is a measure of the voltage V_R across the resistor R. The value of the resistance will be kept constant throughout this part of the experiment. Since Ohm's law holds at all times, a measurement of V_R is also a measure of the current I.

3. Adjust the voltage of the variable transformer to about 10 V.

4. Adjust the sweep time so that you see one or two complete sine waves on the scope face.

5. Adjust the *gain* on the scope so that you can measure the peak-to-peak height easily.

6. Measure and record the peak voltage V_R read from the scope face and record the value of C. Divide V_R by R to find the peak value of the current. The rms current is proportional to this peak current.

7. Turn off the variable transformer, but do not change the setting. This assures the same input voltage and the same setting. Insert an additional resistance about the same size as the first one into the circuit (but not between the scope terminals). Turn the variable transformer back to its original value, and again read the peak

voltage across the original resistor. Has it changed? Again compute the peak current I from the voltage V_R.

8. Turn off the variable transformer, but do not change its setting. This assures the same input voltage. Switch the leads connected to the scope from the resistor to the capacitor. Turn on the output transformer. Record the gain of the scope so you can return to it, then change the gain until the peak voltage can again be read on the scope face. How do the voltage drops measured in steps 7 and 8 compare? Which is larger, or are they the same?

9. Reduce the voltage from the variable transformer until the peak value of the voltage on the scope face is half as large as before this change was made. You are reducing the voltage V_C across the capacitor by a factor of two.

10. Turn off the variable transformer without changing its setting, and switch the scope leads from the capacitor back to the resistor. Restore the gain of the scope to the value it had before the change in step 8. Turn on the variable transformer, and measure the peak voltage across the resistor. How does this voltage compare to the measurement made in step 7 at a higher input voltage? (Be as precise as possible.) Also, compute the peak current I.

11. Again, turn off the variable transformer without changing its setting. Replace the capacitor with another which has larger capacitance. Turn on the variable transformer. Again record the peak voltage across R and the new value of C. How do the results of steps 10 and 11 compare? (Be as precise as possible.) Compute the peak current I.

12. Replace the single capacitor by two capacitors connected in parallel (see Figure 20) and again measure the voltage V_R across the resistor. How does this voltage (current) compare with the sum of the readings taken in steps 10 and 11?

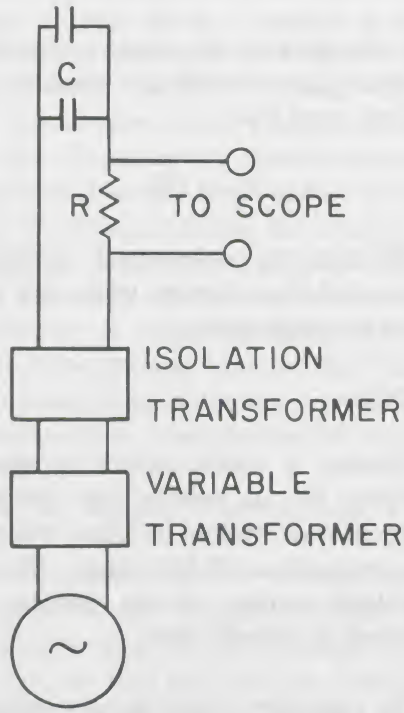

Figure 20.

13. Repeat step 12, but wire the same two capacitors in series this time. Again, measure the voltage V_R across the resistance. How does this voltage (current) compare with the two readings taken in steps 10 and 11?

(Optional)

14. Replace the variable transformer with an *audio oscillator,* which provides a variable frequency source of AC power. Turn on the oscillator and adjust its output amplitude to a value about halfway between zero and maximum. Set the frequency between 30 and 40 Hz. Record the frequency f. Other adjustments may be necessary in order to make measurements on the scope face. Record both the peak value of the voltage V_R across the resistor and the time to complete one complete cycle (in horizontal scale divisions on the scope face).

15. Increase the frequency by a factor of about three and record the new frequency. Again, measure the voltage V_R and the period of a complete cycle in scale divisions. How do these results compare to those obtained in step 14?

Discussion of Part A of Experiment B-1

1. In steps 6 and 7, adding a second resistance probably did not change the current in the circuit noticeably. This implies that the effective circuit resistance, called the *impedance* for AC circuits, was already much larger than the resistance you added. That is, the impedance of the capacitor, called the *capacitive reactance,* must have been large compared to the resistance.

2. You found that the voltage V_C across the capacitor was much larger than the voltage V_R across the resistor. This suggests that, if a relation similar to Ohm's law, such as $V_C = IX_C$, does hold, the proportionality constant X_C, for this case, must be much larger than R, since the current is everywhere the same in a series circuit. This is consistent with the conclusion that the capacitive reactance was much larger than the resistance, if we identify X_C as the capacitive reactance.

3. When the voltage V_C across the capacitor was halved, V_R (and thus the current I in both the resistor and capacitor) was also halved. This is additional evidence that a proportionality relation similar to Ohm's law does hold, at least for peak and for rms voltages and currents, and that the capacitive reactance X_C is a constant characteristic of a particular capacitor in a particular circuit. That is, we suspect that

$$V_C = IX_C$$

In fact, more detailed experiments confirm that this relation is valid. However, we cannot claim that the voltage across a capacitor is *at each instant* proportional to the current through the capacitor, because we did not look at these two quantities simultaneously. We will do this in Part B of the experiment.

4. When you changed the value of the capacitor from C_1 to a larger capacitor C_2, without making any other changes, the current in the circuit increased. What does this mean? Since the voltage output of the variable transformer was the same in both cases, we might suspect that the impedance must have been smaller for the larger value of capacitance. Careful analysis leads to the conclusion that

$$X_C \propto 1/C$$

which shows that X_C is proportional to $1/C$. Let's look at the reasoning. We suspect that, for rms values,

$$V_C = IX_C \tag{7}$$

Because the resistance is so small compared to the capacitive reactance, the voltage V_C across the capacitor is very nearly equal to the source voltage. Since the source voltage did not change, we can argue that the value of V_C did not change appreciably when we changed capacitors. That is, V_C was the same in both steps 10 and 11. Therefore,

$$(V_C)_1 = (V_C)_2$$

or

$$I_1 X_{C1} = I_2 X_{C2}$$

If your measurements were done carefully, they probably indicate that:

$$\frac{I_1}{I_2} = \frac{C_1}{C_2}$$

Combining these two results gives:

$$\frac{I_1}{I_2} = \frac{C_1}{C_2} = \frac{X_{C2}}{X_{C1}}$$

5. When two capacitors were connected

into the circuit in parallel, the voltage V_R was the sum of the voltages V_R measured when each capacitor was connected into the circuit separately. This observation leads to an interesting conclusion. We know that

$$V_R \propto I$$

and

$$I \propto (1/X_C)$$

and

$$X_C \propto 1/C$$

or,

$$V_R \propto C$$

Therefore, the fact that the voltage readings V_R for single capacitors in the circuit add up to the voltage measured when the capacitors are connected into the circuit in parallel means that the capacitance C of C_1 and C_2 connected in parallel is given by

$$C = C_1 + C_2 \text{ (parallel)} \qquad (8)$$

6. You discovered that the current through two capacitors in series is less than the current through either by itself, if the applied voltage is the same in each case. Careful measurements show that

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (series)} \qquad (9)$$

7. If you did the optional experiment, you probably discovered that V_R, and therefore I, is proportional to the frequency f when only the frequency is changed. Again, careful analysis will show that

$$X_C \propto \frac{1}{f}$$

The correct and complete formula for the capacitive reactance X_C is

$$X_C = \frac{1}{2\pi f C} \qquad (10)$$

Since $X_C = (V_C/I)$, the units of capacitive reactance are V/A or Ω. This is consistent with the idea that reactance is effectively a resistance.

Example 2. Compute the capacitive reactance of a 200-μF capacitor for a 60-Hz AC voltage.

Solution.

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \cdot 3.14 \cdot 60 \cdot 200 \times 10^{-6}} \ \Omega$$

$$= 13.3 \ \Omega$$

Example 3. Find the rms voltage across the capacitor of Example 2 if the measured rms current through the capacitor is 0.75 A.

Solution. The rms voltage is given by

$$V_C = IX_C$$

$$= (0.75 \text{ A})(13.3 \ \Omega) = 9.97 \text{ V (rms)}$$

Problem 3. The capacitive reactance of a capacitor is found to be 100 Ω when the applied voltage has a frequency of 30 Hz. Find the capacitance.

Problem 4. The rms voltage across a capacitor is measured to be 40 V while the rms current through the capacitor is 0.13 A. What is the capacitive reactance? Consider the circuit to be a 60-Hz circuit and compute the capacitance.

PART B. Phase

In this part of the experiment, you will determine the phase of the voltage across a capacitor relative to the current through it.

You will need an oscilloscope with a *dual-trace display* capability or an attachment that permits alternate viewing of two voltage signals. (If these are not available, your teacher can show you how to make the

relative-phase measurements with a single-trace scope.)

Background

So far you have compared peak-to-peak voltages across different circuit elements. It is also interesting to compare the phases of different AC voltages. How do we measure a phase difference? Figure 21 shows one complete sine-wave pattern. The curve is called a "sine wave" because it is actually a graph of the sine of angle θ on the vertical axis versus the angle θ on the horizontal axis. For example, the sine of 90° is, from the graph, +1, and the sine of 45° is about 0.7. The complete pattern encompasses 360°. The voltage sine-wave patterns you have been observing are essentially the same as Figure 21. The differences are that the vertical scale is a voltage scale and the horizontal scale is a time scale. This horizontal time scale is related to the angle scale of Figure 21 in a simple way.

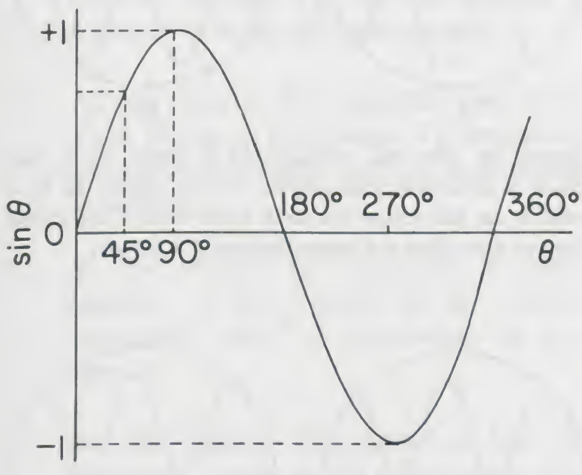

Figure 21.

The amount of time required for a voltage signal to complete one full cycle is called a *period.* One complete period is equal to 360°. It is customary to express phase differences in terms of an angle which is some fraction of 360°. Figure 22 shows two 60-Hz AC voltages on the same time axis. The larger one V_2 reaches its peak at a later time than does V_1. The time elapsed between the peak of V_1 and the peak of V_2 is about 3/8 of the period of a complete cycle. (Note that both voltages have the same period.) We say that the phase of voltage V_2 *lags* the phase of voltage V_1 by 3/8 of 360°, or 135°. Equivalently, we can say that the phase of V_1 *leads* the phase of V_2 by 135°. You will now investigate phase differences between two AC voltages.

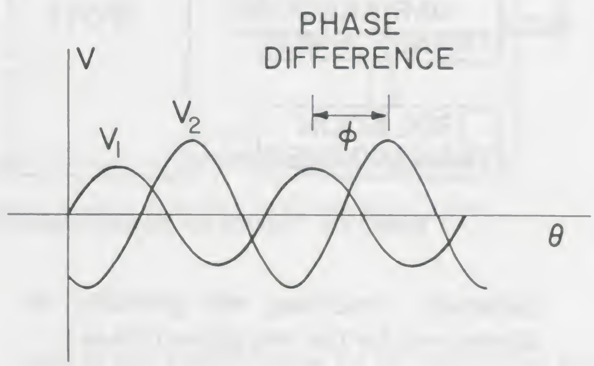

Figure 22. Phase difference.

Procedure

1. Connect a resistor and a *potentiometer* (variable resistor) in series across the output of the variable transformer-isolation transformer combination, as in Figure 23. Use a potentiometer with a total resistance of about 10 kΩ and a resistor of 3 kΩ or so. By connecting only the center tap and one end of the "pot" into the circuit, the value of R_1 can be varied simply by turning the knob of the pot.

2. An easy way to display two voltages and measure their relative phase is with a dual-trace oscilloscope. (Alternatively, a scope attachment called a *display alternator* may be used with a single-trace scope.) The common "ground" connection of the two scope inputs should be connected between the two resistors, as indicated in Figure 23. (It is a common error to connect two leads from each scope input to the circuit. If that is done

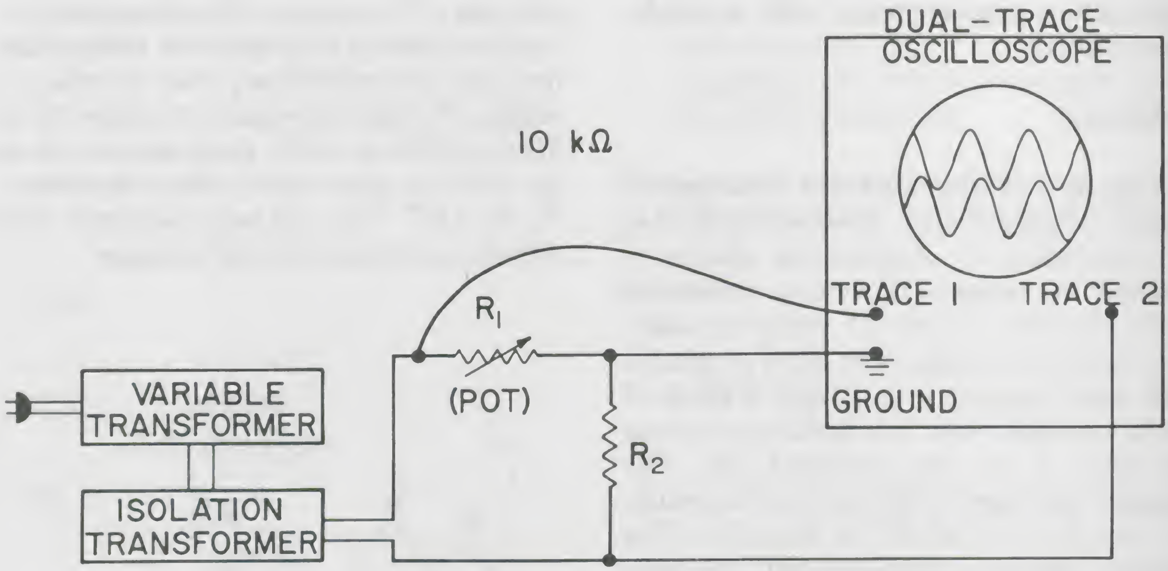

Figure 23. Finding the relative phase of the voltages across two resistors in series.

carelessly, something will probably be shorted out by the two ground leads.)

Set the vertical gain of both traces to the same value, selected so that the complete amplitudes of both traces can be viewed on the scope. Adjust R_1 by turning the pot knob until the two amplitudes are equal. Adjust the sweep time so that slightly more than one complete pattern for each trace can be seen on the scope face.

Because both signals are in phase with the current, they are in phase with each other. But, your scope probably shows them 180° out of phase, as indicated in Figure 24A. This is because of the way the leads from the scope are connected: for one resistor, the signal passes the ground connection *after* going through the resistor; for the other it passes the ground connection *before* going through the resistor. This has the effect of inverting one with respect to the other. If your oscilloscope has a means of inverting one signal, the two may be displayed as in Figure 24B. Otherwise you will have to carefully keep track of phase shifts, as indicated in Figure 25.

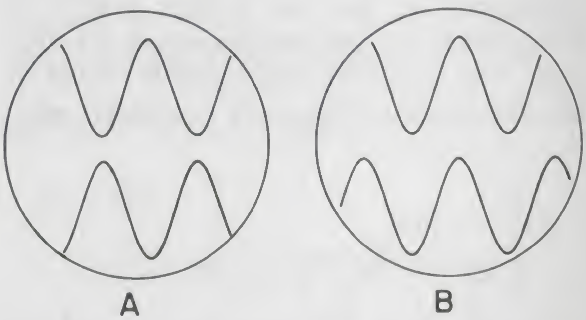

Figure 24. The two voltages are in phase. In A, one trace is inverted with respect to the other. In B, a switch on the scope has been used to flip one signal over so that they are more obviously in phase.

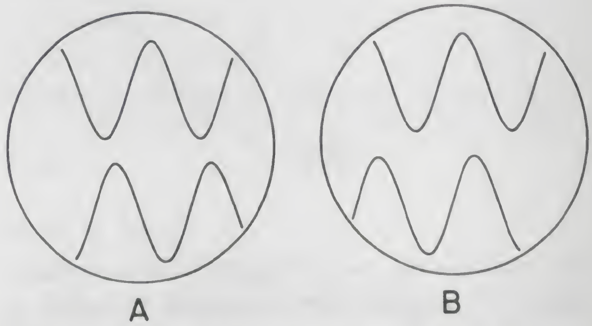

Figure 25. The voltage on the lower trace lags somewhat behind that of the upper trace in the two cases corresponding to Figure 24.

3. Replace the second resistor R_2 with a capacitor whose capacitance is between 0.3 μF and 1.0 μF, and find the phase difference in the two voltages. The circuit is shown in Figure 26.

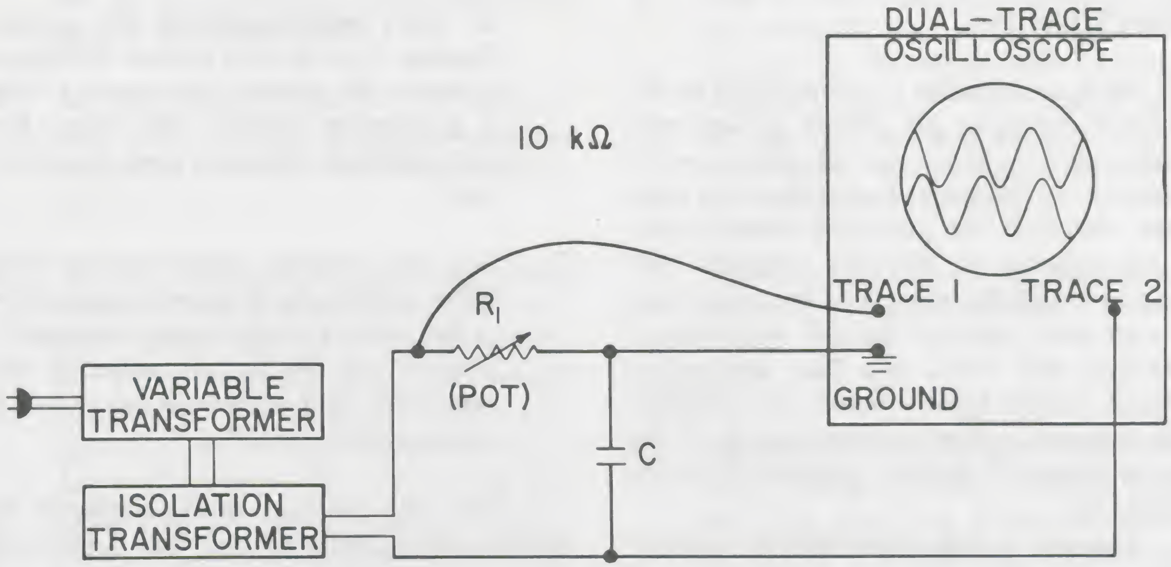

Figure 26. Finding the relative phase of the voltages across a capacitor and a resistor in series.

Questions

1. What is the phase difference between the resistive voltage V_R and the capacitive voltage V_C in an AC series circuit?

2. At the instant the capacitor has maximum charge, is the voltage V_C across the capacitor maximum, zero, or somewhere in between?

3. At the instant the capacitor is fully charged, is the current in the circuit maximum, zero, or somewhere in between?

4. At the instant the capacitor has its maximum charge, is the voltage drop V_R across the resistor maximum, zero, or somewhere in between?

5. Is your answer to Question 1 consistent with your answer to Questions 2 and 4?

Discussion of Part B of Experiment B-1

The fact that the voltage and current across a capacitor are *not* in phase means that the voltage across a capacitor at any given time is *not* proportional to the current in the capacitor at that time. We cannot find an equation like Ohm's law which is valid at each instant. However, $V_C = IX_C$ does hold for peak and rms values.

The answers to the questions above should convince you that the voltage V_C across a capacitor *lags* the current through the capacitor by ¼ of a cycle or 90°. (See Figure 27.)

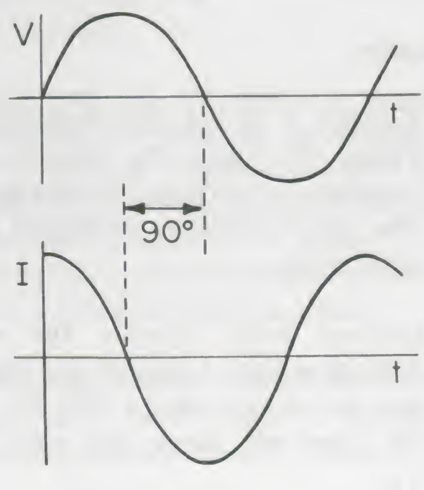

Figure 27.

INDUCTORS

When an inductor is connected in an AC circuit, it carries an AC current, just as a wire resistor does. In fact, since all wire has some resistance, an inductor always has some resistance. However, the important characteristic of an inductor is that the magnetic flux associated with the current in the wire passes through many loops of the coil. According to Faraday's law, when this flux changes, an induced voltage appears across the inductor. This induced voltage is proportional to the rate of change of the flux that links the loops of the coil.

In order to understand how an inductor behaves in an AC circuit, we need to answer the following questions:

1. Is the voltage drop across an inductor at all times proportional to the current through it, or is there a phase difference between the voltage and current? That is, is there an equation like Ohm's law that applies to inductors every instant of time?

2. Does the rms (or peak) voltage drop across an inductor increase in proportion to the rms (or peak) current through it? That is, can we find an equation like Ohm's law that applies to rms inductor voltages and currents?

You can answer these questions by performing Experiment B-2. Part A will help to answer question 2, and Part B will deal with the issue of phases raised in question 1.

EXPERIMENT B-2. Inductors

PART A. Inductive Reactance

The objective of this part of the experiment is to determine if there is a constant X_L for an inductor such that the voltage V_L across the inductor is proportional to the current I through the inductor. That is,

$$V_L = IX_L \tag{11}$$

If such a relation does exist, what does X_L depend on?

Procedure

1. Connect a circuit like that shown in Figure 28. Again, the purpose of the resistor is to allow current measurement. The circuit will be dominated by the inductance.

2. Connect leads between the vertical-deflection-plate terminals on the scope and the resistor (about 100 Ω), so that the scope will display the peak voltage V_R.

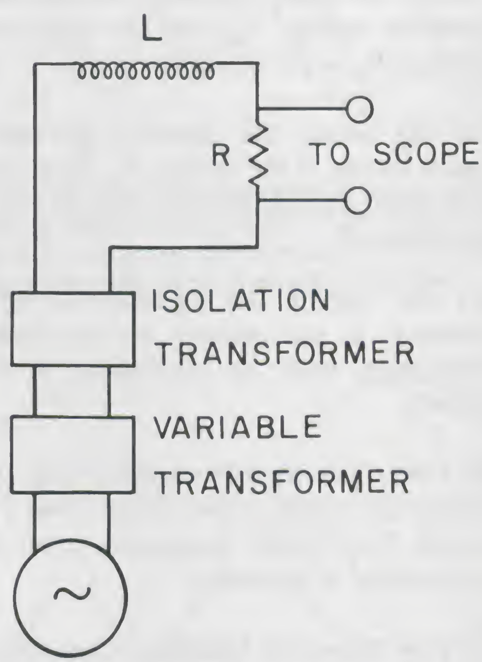

Figure 28. Circuit for Experiment B-2.

3. Adjust the voltage of the variable transformer to about 30 V, the sweep time so that one or two complete cycles appear,

and the gain so that you can measure the peak-to-peak height easily.

4. Record (in scale divisions) the peak-to-peak value of the voltage trace on the scope (V_R) and the value of the inductance L.

5. Turn off the variable transformer. Insert a second resistor (having the same resistance as the first resistor) in series with the other elements, turn the output voltage back to its original value, and again read the voltage V_R on the scope. As in Experiment B-1, if V_R does not change substantially, then you may conclude that the resistance of the resistors is only a small fraction of the total impedance of this circuit.

6. Turn off the variable transformer without changing its setting. Switch the scope leads from the resistor to the inductor. Turn the variable transformer back on. If the scope trace is too large to measure, reduce the gain of the scope (but record its original value so you can return to that setting later). Read and record the peak-to-peak voltage V_L across the inductor.

7. Without making other changes, turn the voltage on the variable transformer down until V_L, as read on the scope face, is half what you measured in step 6.

8. Turn off the variable transformer without changing its setting. Switch the scope leads from the inductor back to the resistor. Increase the gain of the scope to the recorded value (step 6). Turn on the variable transformer. Read and record the new peak-to-peak voltage V_R. If the current I (or the voltage V_R) decreases by half when the inductive voltage V_L decreases by half, then we might guess that the voltage V_L is proportional to the current I. We still need to determine what the proportionality constant X_L depends on.

9. Turn off the variable transformer without changing its setting. Replace the inductor in the circuit with a different inductor. Turn on the variable transformer. Record the new voltage V_R that is displayed on the scope face and the new value of L in the circuit.

   Does V_R (and hence I) increase as L increases, or does it decrease?

Optional

10. Replace the variable transformer with an audio oscillator. Make whatever adjustments are necessary to see and measure the resistive voltage V_R on the scope face. Set the frequency of the oscillator at a value near 90 Hz. Measure and record V_R and the period P for one cycle (in seconds).

11. Reduce the frequency of the oscillator to a value near 45 Hz. Record the new values of V_R and P for one oscillation, in scale divisions. Note that the *inverse* of the period ($1/P$) is the frequency of the alternating current.

    If V_R (and hence I) is proportional to the period, it is inversely proportional to the frequency f. Thus, if nothing but the frequency is changed, we can deduce how X_L varies with f. See if you can do so before reading further.

Discussion of Part A of Experiment B-2

1. You probably discovered, by comparing currents for different resistances, that the resistors contributed little to the impedance of the circuit. Evidently, the inductor contributed most of the impedance. The impedance of an inductor is called *inductive reactance.*

2. Since the current through the resistor is the same as the current through the inductor (always true for series circuits),

and the inductive reactance, which we call X_L, is much larger than the resistance R, then the voltage across the inductor V_L is also much greater than the voltage V_R across the resistor. Your measurements should have convinced you of this.

3. If V_R, and thus I, decreased by half when the voltage V_L across the inductor decreased by half, then you may suspect that the inductive voltage is proportional to the current. Further, you might think that X_L is a constant for a particular inductor in a particular circuit. More careful study would confirm both of these suspicions.

4. If the measured values of peak (and rms) current I decreased as the inductance L was increased, then the inductive reactance X_L is proportional to the inductance L. If so, then we can write (for peak or rms values)

$$V_L = X_L I \tag{11}$$

Since the resistance was very much smaller than the inductive reactance X_L, the voltage V_L across the inductor was very nearly equal to the input voltage. Thus, since the input voltage did not change when you changed inductors, we can say that the voltage across the inductor did not change appreciably. That is,

$$(V_L)_1 \approx (V_L)_2$$

where the subscripts 1 and 2 refer to values of V_L, first with inductance L_1 in the circuit and then with inductance L_2 in the circuit. Then

$$I_1 X_{L1} \approx I_2 X_{L2}$$

If your measurements show that

$$\frac{I_2}{I_1} = \frac{L_1}{L_2}$$

then:

$$\frac{X_{L1}}{X_{L2}} \approx \frac{L_1}{L_2}$$

and X_L is proportional to L.

Your results may not have been very convincing. The fact that an inductor has resistance as well as inductive reactance modifies the arguments. We cannot account for the effect of the inductor's resistance unless we measure it and include it in our questions, which we have not done. If the resistance R_L of the inductor is comparable to X_L, the resistive effects would be quite large. If either the resistor R or the resistance R_C of the inductor is large, the analysis breaks down.

5. If the current I changed in proportion to the change in the period of oscillation as the frequency was changed, we can argue that X_L is proportional to the frequency f.

Although the frequency was changed, the input voltage did not change. Again, since the resistance was very small, almost all of the (constant) input voltage appeared across the inductor. As before,

$$(V_L)_1 \approx (V_L)_2$$

where the subscripts now refer to values of V_L, at frequency f_1 and then at frequency f_2. Thus,

$$(X_L)_1 I_1 \cong (X_L)_2 I_2$$

Since the period (in seconds per cycle) is the reciprocal of the frequency (in cycles per second, or hertz), we know that

$$\frac{I_2}{I_1} = \frac{T_2}{T_1} = \frac{f_1}{f_2}$$

Then,

$$\frac{(X_L)_1}{(X_L)_2} = \frac{I_2}{I_1} = \frac{f_1}{f_2}$$

How convincing were your results? Careful measurements show that the inductive reactance is

$$X_L = 2\pi fL \qquad (12)$$

The units of inductive reactance are ohms (Ω), as for capacitive reactance and resistance.

Example 4. The peak voltage across an inductor is measured to be 100 V and the rms current is found to be 0.7 A. Find the inductive reactance and, if the input frequency is 60 Hz, find the inductance of the coil.

Solution. The peak voltage is related to the rms voltage by

$$V_L \text{ (rms)} = 0.707\ V_L \text{ (peak)}$$
$$= 0.707 \times 100 \text{ V}$$
$$= 70.7 \text{ V}$$

The rms current and voltage across the inductor are related by

$$V_L = I_L X_L$$

or

$$X_L = \frac{V_L}{I_L} = \frac{70.7}{0.7} = 101\ \Omega$$

The inductive reactance is also given by

$$X_L = 2\pi fL$$

Thus:

$$L = \frac{X_L}{2\pi f} = \frac{101}{2 \times 3.14 \times 60} = 0.27 \text{ H}$$
$$= 270 \text{ mH}$$

Problem 5. Calculate the inductive reactance of a 25-H inductor for a frequency of 60 Hz.

PART B. Phase

In this part of the experiment, you will measure the relative phase between the voltage across an inductor and the current through it.

Procedure

1. Connect a potentiometer (10 kΩ) and a large inductor (5-25 H) in series across the output of the variable transformer-isolation transformer combination. Set the variable transformer to about 30 V. (See Figure 29.)

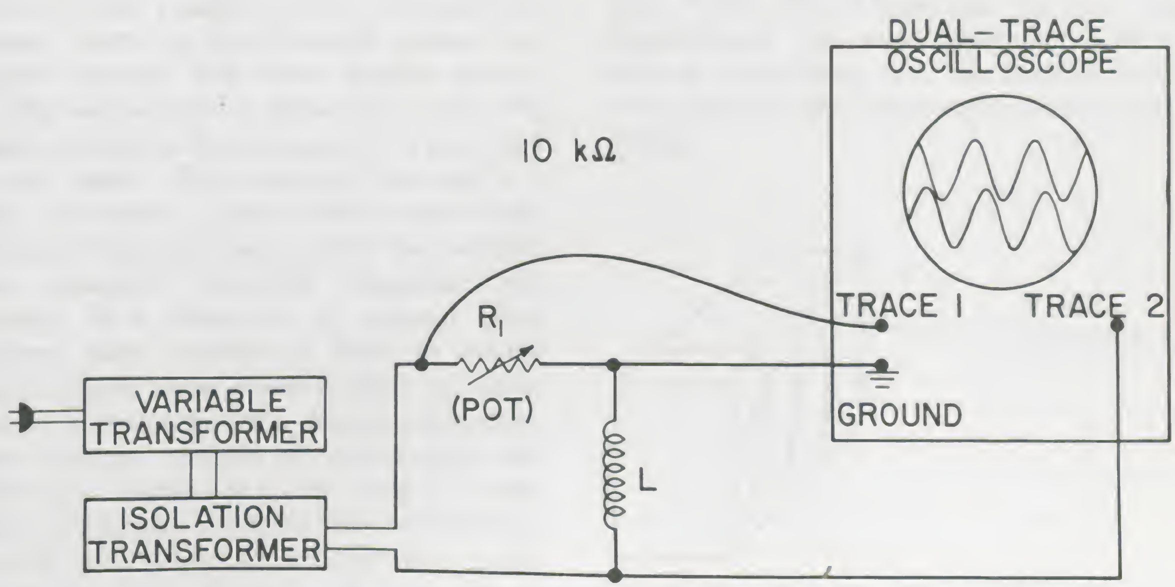

Figure 29. Finding the relative phase of the voltages across an inductor and a resistor in series.

2. As before, use either a dual-trace oscilloscope or a display alternator. Connect the scope leads as in Figure 29. Adjust the potentiometer until both traces have the same height on the scope face. Adjust the sweep time so that just over one complete cycle is visible. Now measure the phase difference between the two voltages.

Discussion of Part B of Experiment B-2

You probably discovered that, for an inductor, the voltage across the inductor *leads* the voltage across the resistor. Thus the inductor voltage leads the current. This is the opposite result from the one you obtained for a capacitor, where you also found that the voltage across a capacitor lags the current through it by exactly 90°. From that, you might guess that, across an inductor, the voltage should lead the current by 90°. This would be a correct guess were it not for the resistance that is an unavoidable part of an inductor. The effect of the resistance is to reduce that phase difference to a value less than 90° (see Figure 30). Is this what you observed?

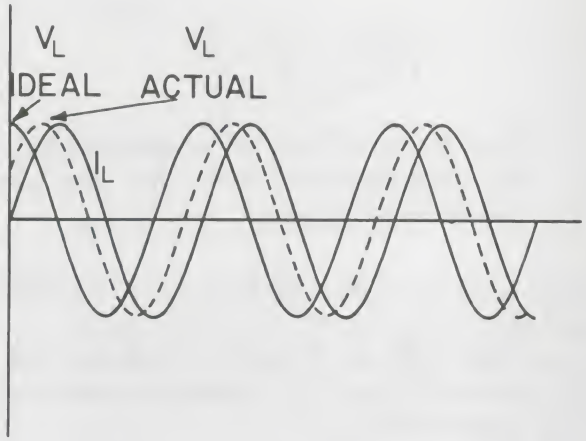

Figure 30. In an ideal inductor—one with no resistance—voltage would lead current by one-quarter of a cycle. In a real inductor, voltage leads current by a lesser amount.

POWER LOSSES

In Experiment A-3 you probably noticed that the core of the laboratory transformer gets quite hot. What are the heat-producing processes in the transformer? We know that the coil has some resistance. Usually, this resistance is small compared to the inductive reactance. Its main effects are to lower slightly the effective voltage of the AC source and to produce a small amount of heat. This heat, which is referred to as *copper loss*, is the same as joule heating in a DC circuit. Two other sources of energy loss are caused by the effects of a changing magnetic field in iron. One of these is called *eddy currents*; the other is called *hysteresis*. Energy which is lost in the form of heat in either of these two ways is called an *iron loss*.

The purpose of a transformer is usually to deliver power to the circuit elements in the secondary. Any losses, such as these iron and copper losses, will waste power. We will examine these loss mechanisms and then we will turn our attention to the power that is delivered to the secondary circuit.

EDDY CURRENTS

At this point, you will have an opportunity to view two short films called *Lenz' Law I* and *Lenz' Law II.* It should be obvious from these films that, when metal objects move through a magnetic field, particularly in regions where the field strength changes, the objects quickly lose their kinetic energy. Careful measurements reveal that, while the kinetic energy is decreasing, heat is generated in the objects. The source of this heat is a type of electric current. What causes these currents? Faraday's law predicts that voltages are generated whenever magnetic flux changes. If a conductor is present, these voltages cause currents to flow. In a large metal object, these currents tend to follow circular paths around the changing flux lines. Such induced currents are called *eddy currents* (see Figure 31). Not only do eddy currents dissipate energy in the form of heat, but the very field that produces them exerts forces on them. Perhaps you have felt or

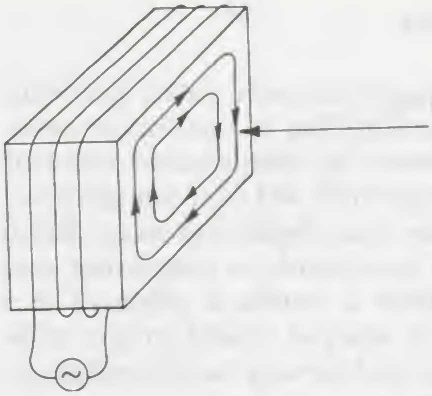

Figure 31. Eddy currents in a solid transformer core.

heard the vibrations of a transformer operating under a load. These mechanical vibrations are caused by the forces the magnetic field exerts on the eddy currents in the transformer core.

The energy dissipated (lost as heat) by the eddy currents in the core of a transformer is supplied by the power source in the primary. This dissipated energy is referred to as *eddy-current loss.* The standard way to minimize this loss is to slice the iron core into thin sheets. Each sheet lies parallel to the direction of the flux lines through the magnetic circuit, and a thin layer of insulation is put between the sheets. Since the eddy-current loops form in planes perpendicular to the flux lines, this slicing procedure—called *laminating*—shortens the length of conductor available to serve as an eddy-current path. (See Figure 32.) Laminating the core cuts eddy-current loss very appreciably. Recall that in Experiment A-1, the laminated iron core produced the largest induced secondary voltage.

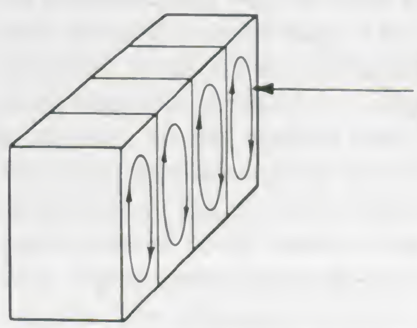

Figure 32. The effect of laminating the core.

HYSTERESIS

Another phenomenon, called *hysteresis,* also causes energy loss in transformer cores. Hysteresis occurs in ferromagnetic materials which are magnetized and then demagnetized. What does the term magnetized mean? Recall your earlier experiments on transformer core materials. When a current is produced in a coil, which is wrapped around an iron transformer core, the following two statements are true. First, the magnetic flux within the coil is much greater than it would be if there were no core material. Second, the magnetic flux within the coil is much greater than it would be if the core were made of a non-ferromagnetic material. Evidently, the flux applied to the iron causes something to happen inside the iron. It causes an additional magnetic flux to be set up within the iron. The magnetic flux produced within the core adds to the flux produced directly by the current in the coil. We say that the core has been *magnetized.* The magnetization is the flux due only to the iron. It is the difference between the total flux and the flux produced by the coil alone.

Since, in this case, the magnetization is caused by a current, you might think that the amount of magnetization is proportional to the current. However, this is not the case. Consider an experiment we could do with a ring of iron with a current winding all the way around it. Initially there is no current and no magnetization. Now gradually increase the current in the coil. Figure 33 shows how the magnetization increases (initially) as the current increases.

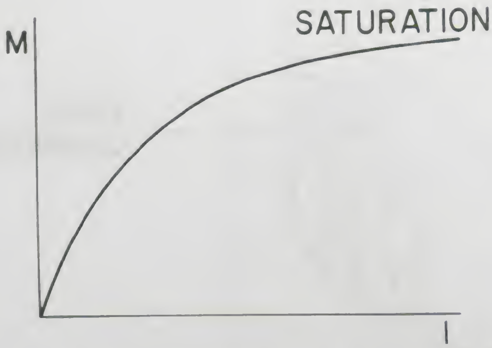

Figure 33. Magnetization versus the magnetizing current for iron.

When the current becomes sufficiently large, the magnetization reaches a constant value. Increasing the current does not increase the magnetization further. Whatever happens in the iron to cause it to set up its own flux has reached an upper limit. At this limit we say the iron core is *saturated.*

Now gradually reduce the current. The magnetization in the iron decreases as the current decreases. The decrease in magnetization as the current decreases is less than the increase in magnetization produced by the increase in current. The "demagnetization" *lags* the magnetization produced by a given current change, as shown in Figure 34. At any given current value, the magnetization is larger when the current is decreasing than when the current is increasing. This phenomenon is called hysteresis and it is a major cause of power loss in a transformer.

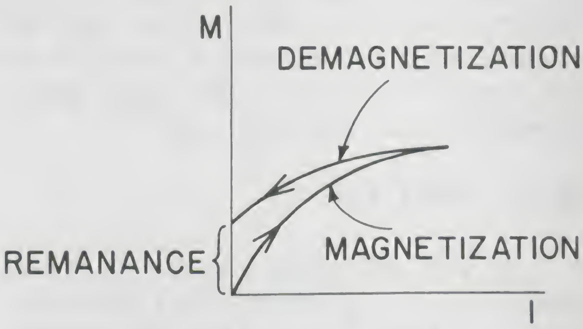

Figure 34.

What happens when the current is turned off? The magnetic flux does not decrease to zero; it has a residual value called the *remanant field* or *remanance.* This remanant flux is "trapped" within the core. The iron core now behaves as a permanent *magnet* without the presence of current. For some materials, such as *hard iron* and some alloys, this residual magnetism is stable against mechanical shock, heating, and the effects of small magnetic fields. For others, such as *soft iron,* the remanance is easily destroyed.*

*The terms "hard" and "soft," when applied to iron, refer to the relative ease with which the material is magnetized and demagnetized. They do *not* refer to mechanical hardness.

If the current is gradually increased in a sense which is opposite to its original direction (a negative current), the magnetization of the iron decreases further and eventually reaches zero (as shown in Figure 35). If the current is increased further in the negative direction, magnetization in the opposite direction develops, and eventually the iron again saturates, but now with the opposite polarity. If the current is now decreased toward zero, this "negative" magnetization is reduced, but again it lags the current. An increasing positive current then produces changes in the magnetization which are symmetric to those already discussed. A complete cycle of the change in magnetization is plotted in Figure 36. This is called a *hysteresis curve.* If the current repeatedly increases and decreases, as an AC current does, a similar magnetization curve is traced out over and over again with each cycle of the current.

Why is hysteresis an energy dissipating process? To answer this question, we need to examine the properties of materials at the atomic level.

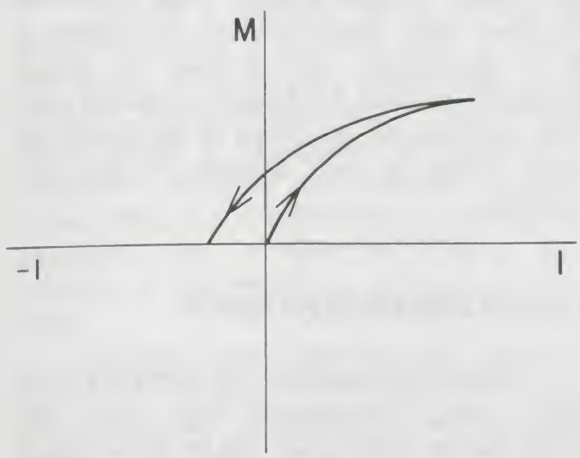

Figure 35.

You may know, from previous study, that breaking a bar magnet into two pieces creates two smaller bar magnets. Breaking these two bar magnets yields four still smaller magnets. The process can be continued, in principle, until the bar magnet has been broken into its constituent atoms. Each of these atoms is composed of electrons "orbiting" around a positively charged nucleus. Each electron "orbit" is a tiny current loop which sets up a magnetic field, much like a large current loop. On a still smaller scale, each electron has "spin." (One useful model is to think of the electron as spinning about an axis, like the earth spins about its polar axis.) This "spin" also effectively produces a current loop, which creates a magnetic field. The strength of each tiny "magnet," either of an orbiting electron or of a spinning electron, is characterized by its *magnetic moment.* The magnetic properties of any particular atom depend primarily on how the individual magnetic moments of its particles add together. The magnetic properties of a substance depend on how the magnetic moments of the atoms add together. These properties fit into three categories: *diamagnetism, paramagnetism,* and *ferromagnetism.*

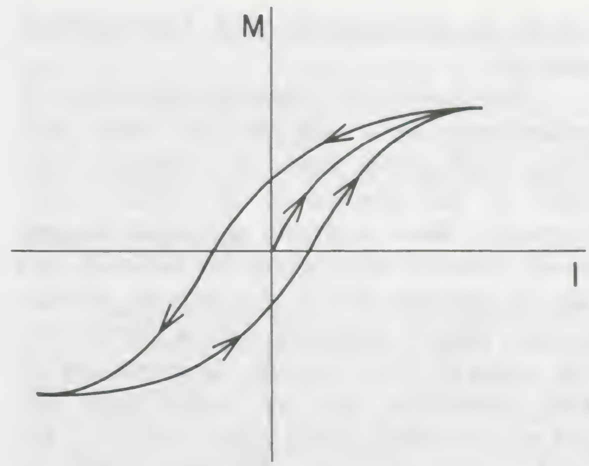

Figure 36. A complete hysteresis curve.

In diamagnetic materials, the individual atoms have zero magnetic moment in the absence of external magnetic fields. An external magnetic field slightly alters the motion of each electron in each atom. The effect of this alteration is to cause each atom to have a small magnetic moment in a direction which is opposite to the applied field. (Figure 37.) This diamagnetism cancels part of the applied field, causing the net field inside the diamagnetic material to be smaller than the applied field. Diamagnetic materials are repelled by a magnet. This diamagnetic effect is present in all materials, but it is overshadowed by larger

effects in paramagnetic and ferromagnetic materials.

The constituent magnetic moments of a paramagnetic atom only partially cancel each other. Such atoms have a net magnetic moment in the absence of an applied field. Normally, these magnetic moments are randomly oriented throughout the material, and the net magnetic effect of a piece of material is zero. When a magnetic field is imposed on the material, the magnetic moments try to align themselves with the field (Figure 38), just as a compass needle aligns itself with the earth's magnetic field. Whatever degree of alignment is achieved strengthens the applied field. However, the random motions associated with the thermal (heat) energy of the material prevent complete alignment. Thus, the amount by which the applied field is strengthened is small.

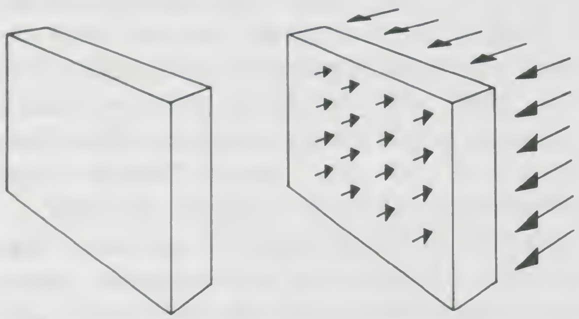

Figure 37. Diamagnetism. The atoms have only induced magnetic moments, which are opposed to the applied field.

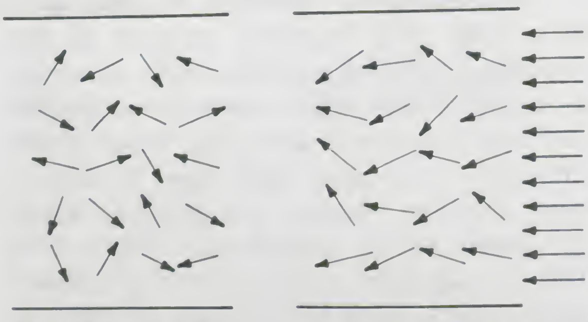

Figure 38. Paramagnetism. The atoms have permanent magnetic moments, which enhance the applied field by partial alignment.

In ferromagnetic materials, the magnetic moments of huge numbers of neighboring atoms interact to align themselves perfectly, even when the material is unmagnetized. These regions of parallel magnetic moments, which are like tiny permanent magnets, are called *domains.* (It is possible to see domain boundaries with the aided eye. Look at the film loop entitled *Ferromagnetic Domain Wall Motion*.) Ferromagnetic materials do not always exhibit large-scale magnetic effects because the domains are usually oriented in random directions. When such a substance is magnetized, the domain boundaries shift in such a way that those domains with magnetic moments parallel to the applied field grow at the expense of others. This means that atomic magnetic moments (or perhaps groups of them) near the domain boundaries must turn to line up with the magnetizing field. Once the magnetic moments are aligned, they tend to stay that way, producing hysteresis effects. Reversing the field is not a loss-free process. When the magnetic moments turn one way and then back again, some energy is converted to the random atomic motions we call thermal energy or heat energy. Thus, hysteresis produces heat, which reduces the efficiency of a transformer. The amount of energy converted to heat by hysteresis during each cycle depends on the shape of the hysteresis curve, which in turn depends on the core material.

TRANSFORMER EFFICIENCY

Unless transformers are designed carefully, various phenomena take place that waste energy. First of all, the magnetic circuit that links the secondary to the primary must be designed to contain as much as possible of the flux produced by the primary coil. While the flux that "leaks" out of the magnetic circuit does not waste energy directly, it does result in a smaller secondary voltage than could be achieved without leakage. Then, in order to deliver the desired energy to the secondary circuit, the primary must operate at higher levels of voltage and current, which results in the generation of more heat and

thus reduces the overall efficiency of the transformer. In practice, engineers have discovered that only iron, nickel, cobalt, and certain iron and nickel compounds called ferrites are capable of carrying large amounts of flux around a magnetic circuit. Even these materials saturate (reach a limiting flux) at high flux levels.

If the iron core is not laminated, large eddy currents are induced in the core, resulting in substantial heat loss, which is totally wasted energy. Laminations limit the size of the possible electrical conducting paths perpendicular to the direction of the flux and successfully reduce eddy-current losses to negligible amounts.

Ferromagnetic core materials also exhibit hysteresis, which involves internal motions of collections of atoms of the core material. These motions, like friction, result in the generation of heat. It is possible to find core materials that exhibit very little hysteresis. Unfortunately, such materials do not carry flux as readily as those that produce more hysteresis loss, and thus a compromise must be made by the designer.

Another potentially serious energy loss in a transformer is the generation of heat in the wires of the circuits, particularly in the primary and secondary coils. These copper losses cannot be eliminated, but they can be minimized by using large-diameter wire. Unfortunately, this solution results in bulky and expensive coils, so again the designer must attempt to determine an optimum compromise.

Another factor that strongly influences the overall efficiency of a transformer is the size and nature of the *load* in the secondary circuit. The load is the component in the secondary circuit to which electrical power is delivered. The transformer efficiency depends in part on how much energy is delivered. If the circuit is well designed, then a large fraction of the power input in the primary will be delivered to the load in the secondary. The fraction of power delivered is defined as the efficiency of the transformer:

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input}} \times 100\% \quad (13)$$

You will have an opportunity to explore how the efficiency of a transformer depends on various design features as you perform Experiment B-3. First, however, we must decide how to measure the power in an AC circuit.

POWER FACTOR

The power delivered to any circuit component is determined by the current through and the voltage across that component. In a DC circuit, the power (in watts) is equal to the current (in amperes) times the voltage (in volts). Although you may be familiar with this relation, it is useful to see how it is derived. By definition, power is the rate of doing work, or the rate of energy flow.

$$\text{Power} = \frac{\text{energy}}{\text{time}}$$

$$P = \frac{E}{t} \quad (14)$$

The potential difference (voltage) between two points, such as the opposite ends of a resistor, is the potential energy lost by charges, as they move from one point to the other, divided by the amount of charge that moves. That is,

$$V = \frac{E}{q} \quad (15)$$

Finally, the electric current in a wire is defined as the amount of charge that flows past a given point in the wire per unit time.

$$I = \frac{q}{t} \quad (16)$$

Combining these definitions, we obtain the known result for power in a DC circuit:

$$P = \frac{E}{t} = \frac{Vq}{t} \quad (17)$$

That is,

$$P = VI$$
(for DC circuits)

It is also true for AC circuits that, at any instant, the (instantaneous) power P is given by the product of the (instantaneous) voltage V and the (instantaneous) current I, or

$$P = VI$$
(for instantaneous AC values)

However, we cannot easily measure instantaneous values of voltage and current at the same time. A more useful quantity is the average power delivered to a component over a period of time. A straightforward, but sophisticated, averaging process leads to the following expression for the average power delivered to an AC component:

$$P_{av} = VI \cos \phi \tag{18}$$

where V and I are the rms values of voltage and current, and the quantity $\cos \phi$ is called the *power factor.* You might suspect that the *phase difference* (ϕ) between voltage and current would play an important role in power dissipation. Equation (18) reflects the fact that it does. The power factor is the cosine of the phase difference.

In a resistor, the current and voltage are always in phase. Thus the phase difference ϕ is zero and $\cos \phi = \cos 0° = 1$. The average power delivered to a resistor is

$$P = VI$$

where V and I are rms values. As in DC circuits, the units of power are watts (W) if voltage is in volts and current is in amperes.

What about a capacitor? In Experiment B-1 you learned that the voltage across a capacitor is 90 degrees out of phase with the current in the capacitor. That is, $\phi = 90°$ and $\cos \phi = 0$. The power delivered to the capacitor is

$$P = VI \cos 90°$$
$$= 0$$

On an average basis, *no* power is delivered to a capacitor. For half of a single cycle, the current and the voltage drop are in the same direction, and energy is delivered to the capacitor. (It is stored in the electric field between the plates.) During the other half of the cycle, the current moves in a direction opposite to the voltage drop, and during this time, energy is delivered to the circuit by the capacitor.

An inductor that has some resistance is a more interesting case. You learned in Experiment B-2 that the phase between the voltage and the current for an inductor is more than zero but less than 90°. This means that for part of a cycle, energy is delivered to the inductor (and some is stored in its magnetic field), and for part of the time, energy is delivered to the circuit by the inductor. However, the amount of time during which the current and the voltage drop are in the same direction is greater than the time that they are in opposite directions. Thus, the net energy delivered to the inductor over a complete cycle is a positive quantity. The power lost in the inductor goes into heating the wire, as in any resistor. If the inductor has a core, it also is heated. An ideal inductor would have a power factor of zero.

Example 5. The rms voltage input to a motor is 30 V, and the rms current is 10 mA. If the phase difference between voltage and current is 30°, compute the power factor and the power consumed by the motor.

Solution. The power factor is

$$\cos \phi = \cos 30° = 0.866$$

and the power delivered to the motor is:

$$P = VI \cos \phi$$
$$= 30 \times 10 \times 10^{-3} \times 0.866$$
$$= 0.26 \text{ W}$$

Problem 6. The rate at which electric energy is delivered to an AC circuit is 500 W. If the rms voltage across the circuit is 440 V and the rms current is 1.5 A, find the power factor of the circuit.

EXPERIMENT B-3. Efficiency of a Transformer

In this experiment, you will measure rms values of the voltage and current for both the load in the secondary and the power source in the primary of a transformer. Then the efficiency of the transformer can be calculated, as well as the phase difference in the primary.

Procedure

1. Connect the circuit indicated in Figure 39. The AC power source should be a 120-V, 60-Hz AC line fed through a variable transformer and an isolation transformer, as in earlier experiments. Use the laminated iron core in the laboratory transformer. Be sure that the primary and secondary coils have different numbers of turns. The small resistor is in the primary only to provide a voltage which is in phase with the primary current. The meters in both primary and secondary circuits are ordinary AC meters. The voltmeters should have scales up to 50 or 100 V, and the ammeters should have scales up to 1 or 2 A. A range of carbon resistors from 10 kΩ to 10 Ω will be used as resistive loads in the secondary. (Light bulbs, 7.5 W to 200 W, work well for the smaller resistances.)

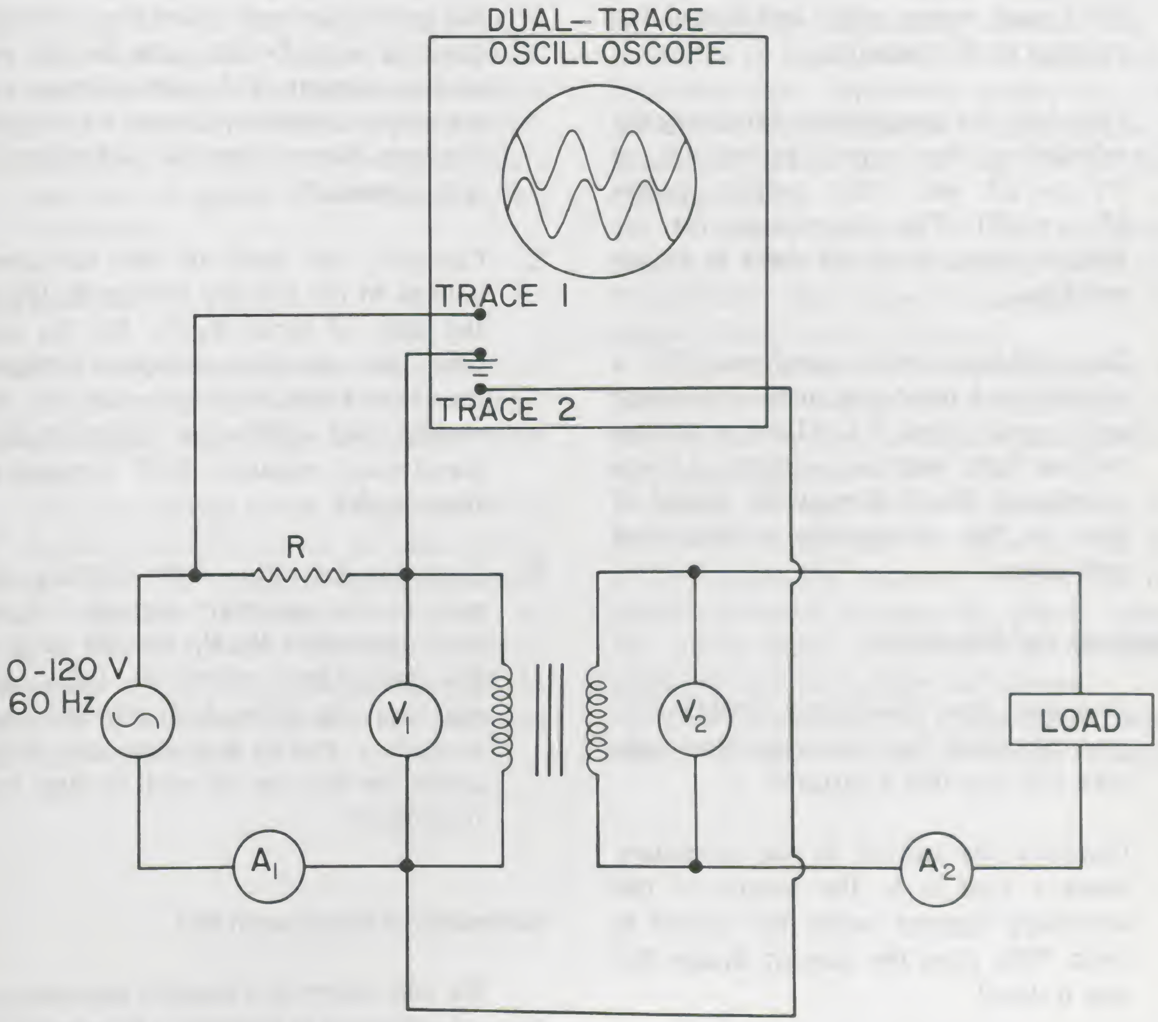

Figure 39. Circuit for Experiment B-3.

2. Plug in the variable transformer and turn up the voltage to 10 V, with the secondary open. Read all four meters and record the readings on the data page.

3. Connect the dual-trace scope, as in your earlier experiments, to measure the voltages across the resistor and the primary coil. The voltage across the resistor is in phase with the current in the primary. Determine the phase angle ϕ between two traces. (Remember that this way of hooking up the scope inverts one trace with respect to the other.)

4. Turn off the variable transformer and place a resistance of about 10 kΩ in the secondary circuit. Turn on the variable transformer and adjust it for the same primary voltage as you had for step 2. Read each meter again and record the readings on the data page.

5. Calculate the transformer efficiency by calculating the input power ($P_{in} = VI \cos \phi$) and the output power ($P_{out} = VI$). The phase angle for the load is zero, since the load is purely resistive.

6. Turn off the variable transformer. Put a smaller load resistance in the secondary and repeat steps 3-5. (Does a smaller wattage light bulb have a higher or lower resistance? Why?) Repeat for a total of four or five successively smaller load resistances.

Questions for Discussion

1. Compare the transformer efficiencies you calculated for the various load resistors. Can you find a pattern?

2. Compare the current in the secondary when a load is in the circuit to the secondary current when the circuit is open. Why does the current change the way it does?

3. Compare the current in the primary when there is a load in the secondary to the current in the primary when the secondary circuit is open.

4. Compare the voltage across the secondary with a small load resistance to the voltage when there is no load. Was there a significant difference?

5. Compare the voltage across the secondary with the voltage across the primary when the secondary circuit is open.

6. Compare the secondary voltage for the case of a small load resistance with the secondary voltage for the case of a large load resistance.

7. Compare the primary current for the case when the load resistance is small to the primary current when the load resistance is large. Do the same for the secondary current. Did both primary and secondary currents change by roughly the same factor when the load resistance was increased?

8. Compare the ratio of the secondary voltage to the primary voltage V_2/V_1 to the ratio of turns N_2/N_1 for the case when the secondary is open. Compare the two ratios for the cases of the various load resistances. Does the ideal transformer equation hold for any of these loads?

9. Compare the ratio of the primary current to the secondary current I_1/I_2 to the turns ratio N_2/N_1 for the cases of the various load resistances. (Why can't you use the ratio I_1/I_2 for the open secondary case?) Are these two ratios closer for the case of small or large load resistances?

Discussion of Experiment B-3

We will return to a detailed discussion of these observations in Section C. However, you may have observed the following:

1. You probably noticed that when the resistance of the load in the secondary was larger, the ratio V_2/V_1 was larger and the currents in both coils were smaller. This leads to smaller losses than when the load resistance is small and the currents large. Because of these losses, a transformer is less efficient when large currents are flowing.

2. For the smaller resistive loads (the larger wattage lamps), you probably found that the ratio V_2/V_1 becomes considerably smaller than the ratio N_2/N_1, especially for the very low-resistance loads, and that the ratio I_1/I_2 is much greater than N_2/N_1 for large-resistance loads and decreases for low-resistance loads.

3. When the secondary was open, there was no current in it. Also, the primary current was smaller with an open secondary circuit than when there was a current in the secondary. For an open secondary, there were few iron or copper losses, but also no energy was delivered to the secondary.

SUMMARY

The rms voltage across a resistor in an AC circuit is proportional to the rms current through it, and the voltage is in phase with the current.

The rms voltage across a capacitor in an AC circuit is proportional to the rms current through it. The proportionality constant, called capacitive reactance, is given by $1(2\pi fC)$, where f is the frequency of current alternation and C is the capacitance of the capacitor. The voltage across a capacitor lags the current through it by 90°.

The rms voltage across an inductor in an AC circuit is proportional to the rms current through it. The proportionality constant, called inductive reactance, is given by $2\pi fL$, where f is the frequency and L is the inductance of the inductor. The voltage drop across an inductor leads the current through it by an angle which is 90° if the resistance of the inductor is very small, and less than 90° if the resistance is larger.

A changing magnetic flux creates small current loops, usually unwanted, which are called eddy currents. If the conducting paths are long, the heat formed in transformer cores by the eddy currents can be quite large. Eddy currents can be largely eliminated by laminating the transformer core.

A ferromagnetic domain contains parallel atomic magnetic moments. When these reverse their direction under the influence of an alternating magnetizing current, heat is produced in the material. This heat is called *hysteresis loss*. Hysteresis results in a lag between domain alignment (magnetization) of the ferromagnetic material and the AC magnetizing current.

A transformer can be used to deliver power to loads in the secondary circuit of a transformer. That power is supplied to the primary circuit by an AC source.

The efficiency of a transformer is defined to be the average power delivered to the load divided by the average power supplied by the source, all times 100%.

The average power delivered to any AC-circuit component is equal to the rms voltage across the component times the rms current through it times the power factor. The power factor is the cosine of the phase angle between the voltage and current.

The efficiency of a transformer changes when the load in the secondary changes.

SECTION C

RESONANCE

One of the most interesting and useful features of an AC circuit has received little attention in this module so far. You probably discovered in Experiment A-2 that the sum of the rms voltage drops around a series AC circuit is greater than the rms voltage across the source of power. You can understand this in terms of your results in Experiments B-1 and B-2. The voltage across a capacitor lags the current through it by 90°, and the voltage across an inductor leads the current through it by an amount that would be 90° if the inductor had no resistance. This means that, even though both the capacitor and the inductor have alternating voltages across them, these voltages are 180° out of phase with each other. (See Figure 40.) Figure 41 shows the instantaneous and rms voltages across an inductor and a capacitor in series. The figure also shows how these two voltages add at each instant of time to produce the voltage sum $V_L + V_C$. (Remember that the rms voltage is 0.707 times the peak voltage.) At any given instant, the voltage across one component is oppositely directed to the voltage across the other. Thus, the total rms voltage across both components is less than the sum of the individual rms voltages.

It is possible for the cancellation of inductor and capacitor voltages to be complete. Figure 42 indicates that this will happen in a series circuit whenever the inductive voltage equals the capacitive voltage. This condition is called *series resonance.* When series resonance occurs, the inductor and the capacitor collectively contribute nothing to the impedance of the circuit. The amount of current is then determined only by the resistance in the circuit. If this resistance is small, the current can be very large. A large current causes large voltages across both the inductor and the capacitor. In fact, these voltages may be much larger than the voltage supplied by the source of power. Such large voltages are good if you want to *amplify* a weak input voltage, such as that picked up by a radio or television antenna. Large resonance voltages can be harmful if you have failed to design your circuit components to withstand high voltages.

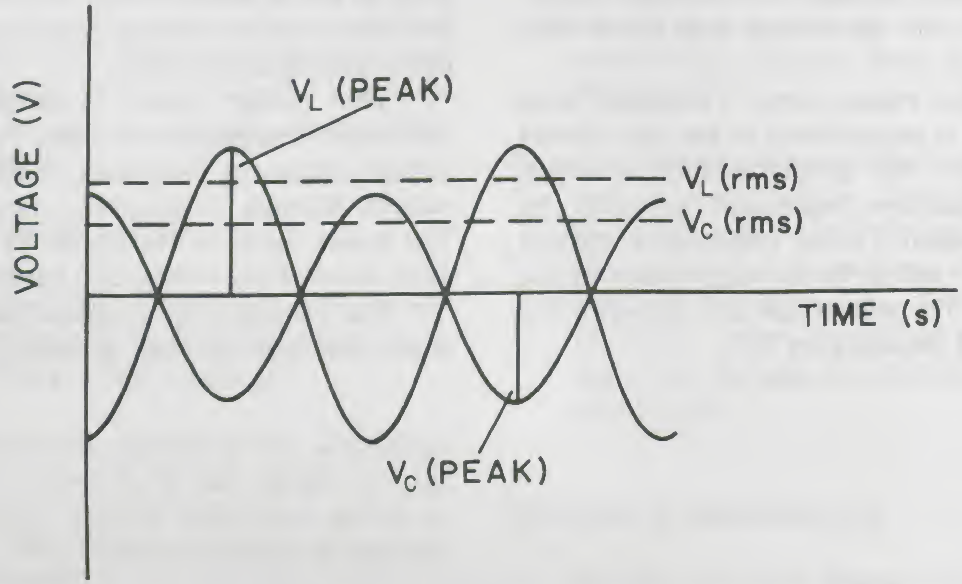

Figure 40. The voltages across an inductor and a capacitor in series have a phase difference of half a cycle.

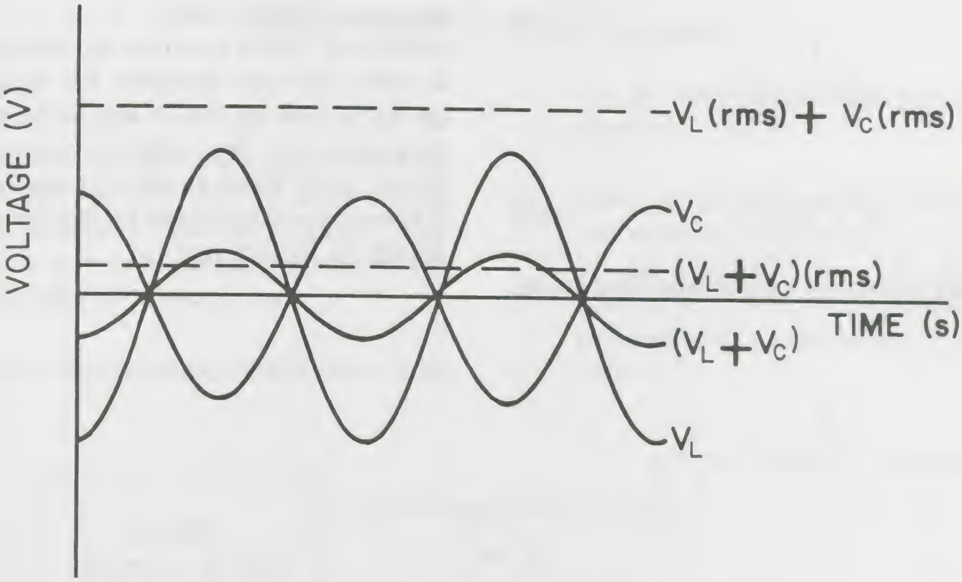

Figure 41. The rms average of the sum of the two voltages is less than the sum of the two individual rms averages.

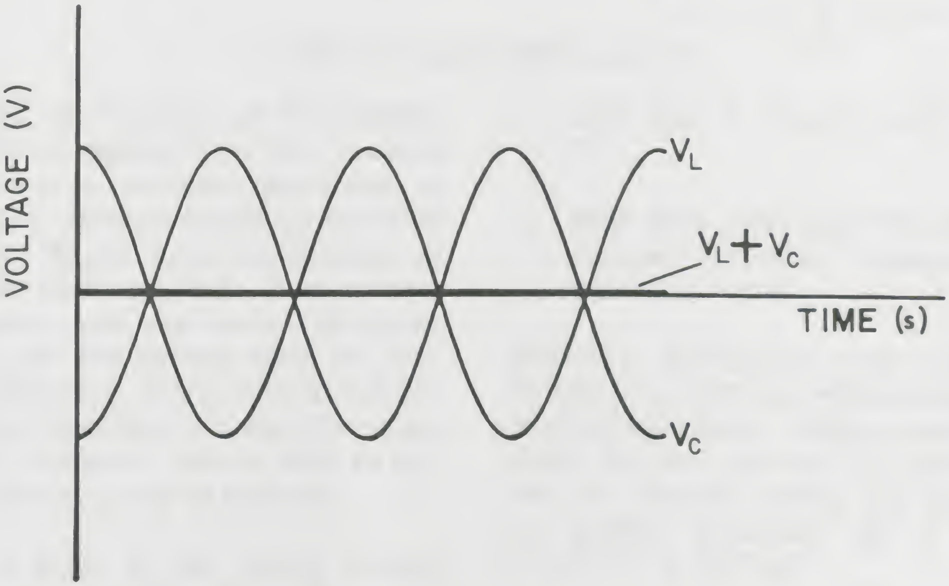

Figure 42. At resonance, the voltages across the inductor and the capacitor cancel.

How does one produce resonance in a series AC circuit? To answer this, we must look at the equation that expresses the condition for resonance:

$$V_L = V_C$$

Since the current is everywhere the same in a series circuit, this implies that

$$X_L = X_C$$

Since

$$X_L = 2\pi f L$$

and

$$X_C = \frac{1}{2\pi f C}$$

we have

$$2\pi fL = \frac{1}{2\pi fC}$$

or

$$4\pi^2 f^2 LC = 1 \qquad (19)$$

For an input voltage at a particular frequency, we can change either L or C to achieve resonance. When you turn the tuning knob on a radio, you are changing the capacitance in an AC circuit so that it will resonate with the frequency of the station you want. In a circuit with fixed L and C, resonance can be achieved by varying the frequency f. You will do this in Experiment C-1.

EXPERIMENT C-1. Series Resonance

Procedure

1. Connect an AC series circuit as indicated in Figure 43. The power source is an audio oscillator. Use an inductor with about a 1-H inductance and a capacitor with about 5-μF capacitance.

2. Calculate the resonant frequency, using the known values of L and C for your circuit.

3. Starting at a frequency of about one-half the resonant frequency, set the output of the oscillator to 1 V. Measure the voltage across the resistor; this voltage is proportional to the current in the circuit.

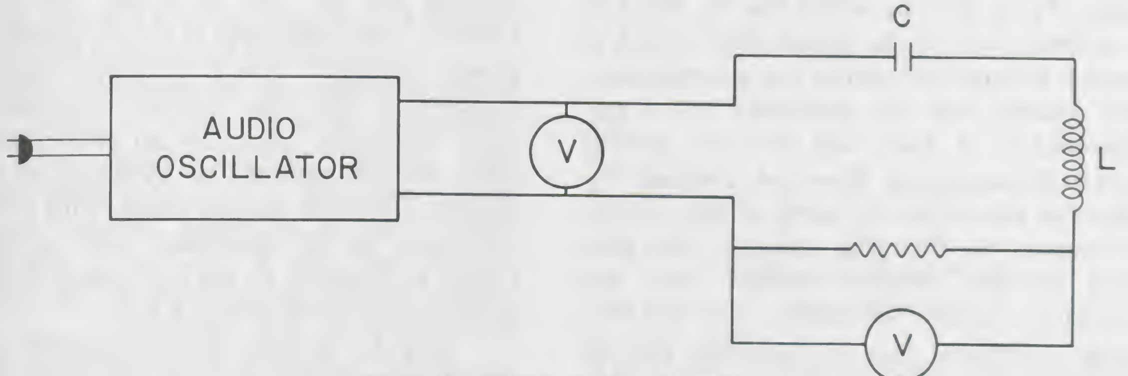

Figure 43. Circuit for Experiment C-1.

4. Increase the frequency of the oscillator by about one-tenth of the resonant frequency and set the output voltage to 1 V again. Measure the voltage across the resistor. Repeat these measurements in steps of about one-tenth of the resonant frequency until you reach a frequency about one and one-half times the resonant frequency. As you pass through the resonant frequency, you may have to use smaller frequency steps in order to find the exact frequency of resonance.

5. Plot a graph of the current (voltage across the resistor) versus the frequency.

Questions

1. Examine your curve of amplitude of the current versus the frequency. How do the values of the smallest and largest currents you observed compare? How rapidly did the current amplitude increase when it was near its maximum value? Calculate the effective impedance of the LRC circuit at the resonant point. How does it compare to the resistance R?

2. How does your measured value of the resonant frequency compare to the theoretical value?

Problem 7. Assume that a radio tuning circuit consists of an inductor with an inductance of 50 μH in series with a variable capacitor. What should be the value of the capacitance in order to "tune in" a station that broadcasts at its assigned frequency of 3 megahertz ($3 \text{ MHz} = 3 \times 10^6$ Hz)?

Problem 8. Find the resonant frequency of the series circuit shown in Figure 44.

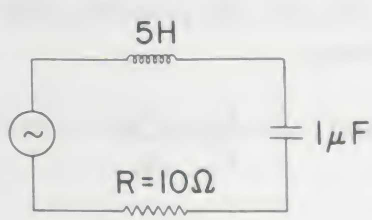

Figure 44.

TRANSFORMER EFFICIENCY REVISITED

Can you use the knowledge and experience gained in this module to understand the results of Experiment B-3 more clearly? We will discuss one by one how various factors influence transformer performance. First, we will consider the behavior of a highly idealized transformer. Then we will introduce the complicating factors one at a time.

We initially assume that the full source voltage V_1 is directly across the N_1 turns of the primary coil of the transformer, which is assumed to have inductance but no resistance. Also, assume that the secondary coil is not connected to a load, and thus no current exists in the secondary. However, a voltage V_2 does exist across the N_2 turns of the secondary because the changing magnetic flux produced by the primary current links the secondary. Finally, we assume that this flux linkage is perfect, that no hysteresis and no eddy currents exist in the iron core, and that the secondary has inductance but no resistance.

Under such highly idealized conditions, the flux change per unit time is the same through every separate turn in both the primary and the secondary. Then, according to Faraday's law, the induced voltage across every turn is also the same. Since the turns are connected in series, the total induced voltage V_1 across the primary is proportional to the number of primary turns N_1, and the induced voltage V_2 across the secondary is proportional to N_2. That is,

$$V_1 \propto N_1$$

$$V_2 \propto N_2$$

Since the rate of change of flux in the two coils is the same, the proportionality relations are the same.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad (20)$$

What is more, these two voltages are in phase with each other since the same flux changes induce both. But both voltages are 90° out of phase with the flux itself. The flux is directly proportional and in phase with the current in the primary circuit, provided that we do not saturate the iron in the transformer core. Thus, the flux is in phase with the current in the primary, which is 90° out of phase with the voltage.

Since the current and the voltage in the primary are 90° out of phase, the power factor is zero and there is no net delivery of power anywhere. This is consistent with the assumption that the primary resistance is zero. Therefore, there are no copper losses. Since we also assumed no hysteresis or eddy current, there are no iron losses. With no load connected to the secondary coil, no useful energy is delivered to the secondary. In other words, not much is going on yet.

Now let us relax the assumption about perfect flux linkage. Suppose that some of the flux produced by the primary coil leaks out of the magnetic circuit and fails to pass through the turns of the secondary coil. However, smaller flux changes through the N_2 turns of the secondary induce a smaller voltage in the secondary than would be the case for no leakage. As a result,

$$\frac{V_2}{V_1} < \left(\frac{V_2}{V_1}\right)_{\text{Ideal}} = \frac{N_2}{N_1}$$

(when there is flux leakage)

This does not directly reduce the efficiency of the transformer, because no energy is wasted by leaking flux. However, if the transformer is designed to step up the primary voltage, then its purpose is not realized as well when there is flux leakage. With proper design and the use of good core material, flux leakage can be made negligible. Therefore, we will assume in what follows that no leakage will occur.

Now suppose that the secondary coil is connected across a load and delivers power to the load. The power factor in the secondary is

not zero, otherwise no power could be delivered. In fact, if the load is a "pure" resistance, then the current through the load is in phase with the voltage across it, and the power factor is 1. Since the load is connected directly across the secondary coil, the voltage drop across the secondary coil is the negative of that across the load. (Remember Kirchhoff's laws?) The same current passes through both. Thus the secondary-coil current and voltage drop are 180° out of phase. The power factor (cos 180°) is −1 and the secondary coil acts like a source (similar to a battery, except that it delivers AC current).

What happens in the primary circuit when a load is connected in the secondary? The power factor for the primary must change from zero. In fact, the *phase* of the primary current shifts to create a phase difference ϕ between it and the primary voltage. We can separate this primary current into two components, one of which is in phase with the primary voltage. These two components, which add together to produce the total current, are 90° out of phase with each other (see Figure 45). The portion of the current which is in phase with the primary voltage ($\cos \phi = 1$) is responsible for the power consumption by the primary. The other part of the current is 90° out of phase with the primary voltage ($\cos 90° = 0$) and in phase with the primary flux. This "flux current" component uses no power. If the "power current" is much larger than the flux current, the phase difference between primary voltage and current is small and the power factor is approximately 1. Under such conditions,

$$V_1 I_1 \approx V_2 I_2$$

and

$$\frac{I_1}{I_2} \approx \frac{V_2}{V_1} \approx \frac{N_2}{N_1} \tag{21}$$

(when power delivery is near optimum)

If the power delivery is small, the two parts of the primary current may be comparable in size. The phase difference ϕ between primary current and voltage then increases, and the

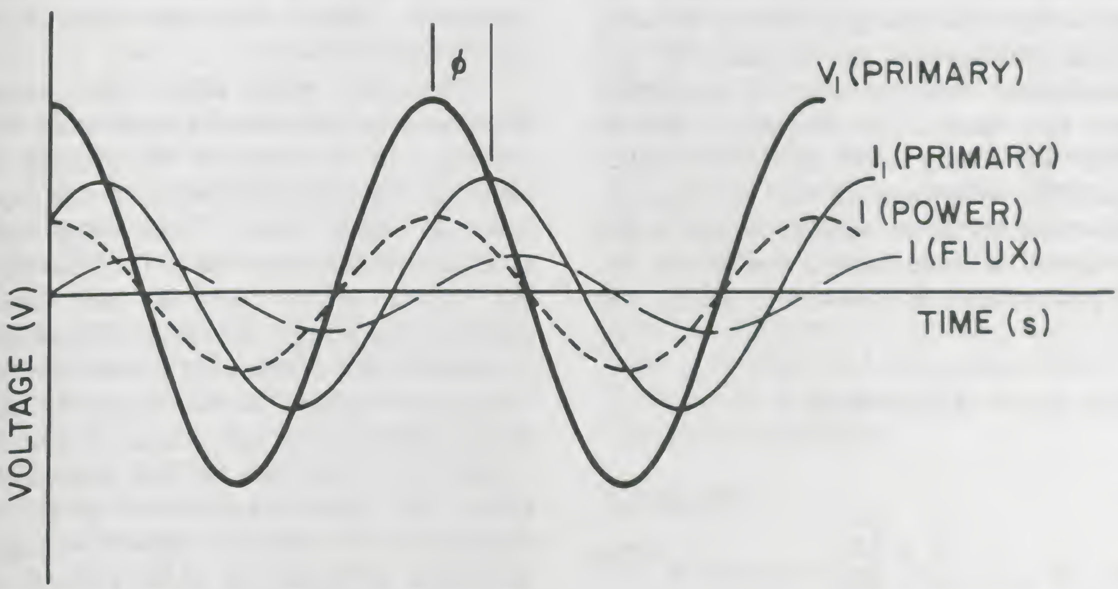

Figure 45. The net current in the primary may be considered to be the sum of the "power current," in phase with the applied voltage, and the "flux current," 90° out of phase.

power factor decreases accordingly. Thus, even if the transformer efficiency were 100%, it would be true that

$$V_1 I_1 \cos \phi = V_2 I_2$$

This means that:

$$V_1 I_1 > V_2 I_2$$

and

$$\frac{I_1}{I_2} > \frac{V_2}{V_1} \qquad (22)$$

(when power delivery is small)

As long as the input voltage is kept constant, the part of the primary current that is in phase with the flux does not change its size as the power current changes. The maximum flux in the magnetic circuit increases as the flux current increases.

In any real transformer, there are also energy losses as a result of hysteresis and eddy currents in the core—the so-called iron losses. These losses increase with the magnitude of the flux, which in turn is proportional to the input current. The heat generated in the iron by these effects must be supplied by the power source. That is, some of the power current that flows in the primary is used to produce this energy in the iron. This further reduces the efficiency of the transformer. Thus, even if the power delivery is large, if the flux current is insignificant, and if the primary power factor is close to 1, it will be true that

$$V_1 I_1 > V_2 I_1$$

and

$$\frac{I_1}{I_2} > \frac{V_2}{V_1} \qquad (23)$$

(when there are iron losses)

Fortunately, these losses can be minimized by proper design of the transformer. Laminating the core virtually eliminates eddy currents, and hysteresis can be made small by choosing a suitable core material.

Another problem in the design of a transformer core is saturation. Experiments show that the reluctance depends inversely on the cross-sectional area of the core. If the cross-sectional area of the magnetic circuit is not sufficiently large, the flux produced by the alignment of dipoles in the iron may reach a limit that is characteristic of the iron, after which it no longer increases as the current increases. Then, if the input voltage of the source is raised further, only the flux produced directly by the current continues to increase. In effect, saturating the iron core reduces the inductance of the coils. This means that a given increase in voltage produces a greater increase in the flux-producing current after saturation than the same change in voltage produced before saturation. Then the phase of the net primary current is shifted farther away from the phase of the voltage across the primary coil, and this further reduces the primary power factor. Thus, for a given increase in input power, the available power does not increase as rapidly after saturation as it does before the iron is saturated. This loss of efficiency can be an important effect, and saturation should usually be avoided.

The only other effect that strongly influences the behavior of a transformer is the resistance in the wires of the circuits. The heat that is generated in the wires—the copper losses—is energy wasted. Such losses cannot be avoided completely. Since they increase as the square of the current, they become relatively more important than the iron losses as more power is delivered to the load at a fixed input voltage. To the extent that most of the resistance in both primary and secondary circuits, other than the load resistance, is that of the coils, the effect of greater coil resistance is to require a greater voltage to produce a given current in the primary coil, where the voltage is in phase with the power current. If, on the other hand, the input voltage is held constant, this results in a decrease in the voltage across the secondary coil, where the voltage is 180° out of phase

with the current. The result is a decrease in the efficiency of the transformer at higher currents. This is consistent with the fact that energy is wasted by the copper losses.

Now let us see how these considerations help us to understand the results of Experiment B-3. For a low-resistance secondary load, you probably found that the ratio V_2/V_1 was substantially less than the ratio N_2/N_1, and the ratio I_1/I_2 was probably a little larger than N_2/N_1. Since this was a low-resistance load, the currents were relatively large. The fact that I_1/I_2 was only a little larger than N_2/N_1 at high currents is an indication that the power current in the primary was much larger than the flux current. However, high currents do lead to substantial copper losses, which are proportional to the square of the current. Copper losses also account for the fact that the efficiency you calculated was less than 100%.

For the case of the high-resistance secondary load, where small currents were flowing, you probably found that V_2/V_1 was nearly equal to N_2/N_1, indicating that the flux linkage was good. You also may have found that the ratio I_1/I_2 was substantially larger than N_2/N_1. This is in agreement with the fact that, at small currents, the power current is small in comparison to the flux current. Notice that, as the secondary resistance becomes larger and larger, we approach the case where the secondary is open.

With the secondary open, you observed that the ratio V_2/V_1 increased to the value N_2/N_1. The primary current dropped substantially as soon as the secondary opened. The current that remained was the flux current (except for the small current that supplied the copper losses in the primary and the iron losses in the core), and thus the phase angle between the current and output voltage increased markedly. Even though the output voltage approaches that for the ideal transformer when the secondary is open, the efficiency of the transformer is zero, since there is no output power.

Although different transformers are designed to do many different jobs, and thus may have very different operating characteristics, they all share some common properties. You have seen some of them in Experiment B-3 and in the discussion above. For instance, when the current is much larger than that for which the transformer was designed, the losses (especially the copper losses) will reduce the efficiency of the transformer. You also saw when the transformer secondary circuit was open that transformers have iron and copper losses even when no output power is being delivered. Because of these losses, the transformer's efficiency will also be reduced when it operates at currents lower than those for which it was designed. In that situation, the output power is so low that it may be comparable in size to the losses that occur at low currents.

You should be able to see from your data in Experiment B-3 that, as the output power grows, the efficiency improves for a range of load resistances (because the losses don't increase as rapidly). But then, as the current increases further with still smaller load resistances, the copper losses (which increase as the square of the current) begin to grow more rapidly, and the efficiency goes down. You may even have felt the heat produced by wasted power. Depending on the transformer which you used, you may or may not have been able to see the full range of this behavior but you should have been able to see at least a part of it.

Transformer design is a very important part of our technology—from the large power transformers that help to deliver power to our homes and industries, to the small audio transformers in your hi-fi amplifier. Each one must be designed to minimize losses in the range of voltages and currents in which it will be used, and at the same time its cost must be held as low as possible.

SUMMARY

Resonance exists in a series AC circuit whenever the capacitive reactance equals the inductive reactance. When resonance occurs, the current, which may be large, is determined only by the resistance and the voltage. Resonance can be achieved for a given fre-

quency by altering the capacitance, the inductance, or both. For fixed values of inductance and capacitance, resonance can be achieved by changing the frequency.

In an ideal transformer that has perfect flux linkage and 100% efficiency, and that draws a total current that is large compared to the current which produces the flux in the transformer core,

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}$$
(ideal case)

As the power current in the primary becomes comparable to, and even larger than, the flux current, the power factor in the primary increases.

Flux leakage and saturation of the core iron reduce the effectiveness of the transformer in the sense that not as much power can be delivered to a load for a given input voltage at the source.

Hysteresis and eddy currents in the iron core produce wasted heat known as iron losses. These increase as the flux in the core increases, which in turn increases as the input current increases.

Resistance in the wires of the circuits also causes wasted heat, known as copper loss, to be generated. This wasted heat increases with the square of the power current.

Both iron losses and copper losses result in reduced output voltage in the secondary, and both reduce the efficiency of the transformer. The reduction in efficiency is usually harder to eliminate. The efficiency changes as the resistance of the load changes and the power delivered to the load changes.

APPENDIX: The Oscilloscope

The oscilloscope is nothing more than a very fast "graphic machine." It is able to present graphs of voltages (usually on the vertical axis) against time or other voltages (on the horizontal axis). Thus the scope displays the size, shape, and frequency of an input signal. How the oscilloscope circuits are designed is not important for this module. Instead, we shall examine the functions of the external controls which are common to most general-purpose laboratory oscilloscopes. Because most oscilloscopes have additional features and because the naming of controls varies among oscilloscopes, the operating instructions for a specific oscilloscope should always be consulted before use. (The terms scope and oscilloscope are used interchangeably here.)

Some of the basic controls found on all oscilloscopes are listed below, along with a brief explanation of their functions. As you read the description of each control, you should refer to the one on your scope.

I. Screen Section

A. The AC ON/OFF control turns the oscilloscope on. It is usually combined with either the INTENSITY control or the SCALE ILLUMINATION control. Most scopes have a panel light to indicate when they are on.

B. The INTENSITY control changes the brightness of the spot (or trace). If a bright spot is left in one place on the screen for too long, it can "burn" away the phosphor on the screen, leaving a permanent dark spot. For this reason, the intensity is usually kept as low as is consistent with good viewing.

C. The FOCUS control adjusts the sharpness of the trace. It should be adjusted to give the narrowest line possible.

D. The VERTICAL and HORIZONTAL POSITION controls change the up-down and left-right positions of the beam. On some scopes, you may find a BEAM FINDER button, which allows immediate location of a beam which is "off the screen." After finding the beam with the beam finder, you can then use the positioning controls to move the beam to the center of the screen.

II. Vertical Amplifier Section

E. The voltage to be analyzed is usually connected to the VERTICAL INPUT. If there is a *probe* provided with the scope, be sure to check its *attenuation factor.* Most probes are designed to reduce the voltage input to the scope by a factor of ten or more. Any calculation of voltage should include this factor.

F. The VOLTS/CM (also called VERTICAL SENSITIVITY) control allows measurement of the size of a voltage signal. The face of most oscilloscope screens is covered with a grid which is ruled in centimeters. This grid is used to read the voltage of a signal. For example, suppose the trace is 2.5 cm high and the VOLT/DIV is set on 10 VOLTS/CM. Then the voltage input to the scope is 2.5 cm × 10 VOLTS/CM or 25 V. (On many scopes, a concentric control knob allows the vertical sensitivity to vary. For calibrated readings this knob must be properly set.)

III. Time Base Section

G. The TIME/CM control (also called the SWEEP RANGE SELECTOR) varies the rate at which the trace "sweeps" across the oscilloscope screen. This feature is necessary to allow measurement of the duration

of a signal or of the frequency of the signal. For example, if a voltage pulse is 4 cm long and the TIME/CM switch is set on 20 ms/cm, the time required for the pulse is 4 cm X 20 ms/cm or 80 ms. (On many scopes a *multiplier* switch allows the calibration time to be changed by some factor.)

H. A SYNC (synchronization) control is part of the time base section on many inexpensive scopes. Its purpose is to cause the sweep voltage to always begin at the same point of a recurrent signal so the signal appears to "stand still." The SYNC control is turned until the pattern on the screen appears stationary. If you cannot "stop" the signal, you may need to choose a different sweep rate.

I. The SYNC SELECTOR control selects one of three possible *modes of synchronization.* On INTERNAL SYNC, the sweep of the scope is synchronized with the input signal. On EXTERNAL SYNC, the sweep is synchronized with an external signal; on LINE SYNC, the sweep is synchronized with the AC line voltage. This control is usually set to INTERNAL.

J. On more sophisticated scopes, the TRIGGER controls achieve synchronization. Triggering means that the horizontal sweep is caused to begin (triggered) whenever the input signal reaches a predetermined value.

The following exercise is to acquaint you with the controls of the oscilloscope you will use in the *Transformer* module. You will need to consult the operating manual for your scope. If you have problems, consult your instructor.

LABORATORY EXERCISE

1. With the power off, examine the scope and locate the following controls or their equivalents.

   a. INTENSITY

   b. FOCUS

   c. VERTICAL INPUT TERMINALS

   d. VOLTS/CM

   e. TIME/CM

   f. SYNC controls or

   g. TRIGGER controls

2. a. Find out, from the operating manual, how to turn off the internal sweep. This may be done by setting the TRIGGER LEVEL to a high position, or by turning the sweep selector to EXTERNAL (with no external input signal).

   b. Turn the FOCUS and INTENSITY controls fully counterclockwise.

   c. Set the VERTICAL and HORIZONTAL POSITION controls to the midpoints of their positions.

   d. Turn on the scope and allow it to warm up for at least one minute.

   e. Turn up the intensity until you see a spot on the screen. Do not turn the intensity higher than you need to. A glow around the spot indicates a too-high intensity. If no spot is visible, it is probably "off screen." Use the positioning controls to move the spot onto the screen.

   f. Rotate the FOCUS control to see

the effect, and adjust to get a very small bright dot on the screen.

g. Observe the effects of the HORIZONTAL and VERTICAL position controls and center the dot.

h. Set the SWEEP selector to an internal sweep and turn on a recurrent sweep. You will need to consult the operating instructions.

i. Observe the effect of changing the TIME/CM setting. Expand the dot into a line by increasing the sweep rate.

3. You are now ready to observe an input signal.

a. Connect the voltage output of a sine- and square-wave generator to the vertical input of the scope. Connect the ground terminal of the generator to the ground input of the scope.

b. If you have a triggered scope, set the TRIGGER to AUTO. If the scope is not triggered, set the SYNC selector to INTERNAL.

c. Set the TIME/CM switch to the slowest sweep rate (on the order of 1 s/cm). (Make sure the vertical is set for *calibrated* measurements.)

d. Set the generator to give a sine wave with a frequency of 1000 Hz. Turn on the generator.

e. Adjust the generator output (and/or VOLTS/CM control) to obtain a signal which is about 5 cm high.

f. Observe the effect of increasing the sweep rate.

g. Observe the effect of varying the frequency of the input signal.

h. Change the output of the generator to "square wave" and observe the pattern.

i. If you have a triggered scope, set the TRIGGER to INTERNAL. Adjust the TRIGGER LEVEL control until you get a stable trace which "stands still." Center the trace.

j. Measure the length of a single pattern on the screen and use the TIME/CM setting to compute the duration of the pattern. How does your measured value compare with the value computed from the known input frequency? (Be sure the *multiplier* is set to 1, or that you include the proper factor.)

k. Change the input frequency and repeat (i) and (j).

DATA PAGES

EXPERIMENT A-1. Basic Transformer Characteristics

1. ________ primary turns

   ________ secondary turns

2-8.

	2	3	4	5	6	7	8
Primary voltage (V)							
Secondary voltage (V)							

9.

Questions

1.

2.

3.

EXPERIMENT A-2. A Simple AC Circuit

3. Variable transformer setting = ________ V

   Peak-to-peak primary voltage = ________ scale divisions

   = ________ V = scale divisions × V/cm

4. Peak-to-peak resistor voltage = ________ scale divisions

   = ________ V

5. Peak-to-peak voltage across resistor = ________ V

   capacitor = ________ V

   capacitor = ________ V

   inductor = ________ V

   inductor = ________ V

6. Peak-to-peak voltage across series resistors = ________ V

8. Battery voltage = ________ V = number of scale divisions ________ × V/cm

Questions

1.

2.

3.

4.

5.

6.

EXPERIMENT A-3. Measurement of Reluctance

1. Primary current I_1 = ________ A

2. Secondary voltage V_2 = ________ V (no gap)

4-5.

Number of shims n	1	2	3	4									
Secondary voltage V_2 (V)													

6. Graph of $1/V_2$ versus n

Questions

1.

2.

3.

4.

5.

EXPERIMENT A-4. Turns Ratio

1. $N_1 =$ ________ turns, $N_2 =$ ________ turns

2.

Primary voltage V_1 (V)	3	6	9	12	15	18	21	24	27	30
Secondary voltage V_2 (V)										

3. Graph of V_1 versus V_2

4. $N_1 =$ ________ turns, $V_1 = 15$ V

Secondary turns N_2				
Secondary voltage V_2				

5. Graph of V_2 versus N_2

6. $N_2 =$ ________ turns, $V_1 = 15$ V

Primary turns N_1				
Secondary voltage V_2				
$1/N_1$				

7. Graph of $1/N_1$ versus V_2

EXPERIMENT B-1. Capacitors in AC Circuits

PART A. Capacitive Reactance

1. $R =$ ________ Ω

6. Variable transformer setting = ______ V

Peak voltage $V_R =$ ________ V

$C =$ ________ μF

Peak current $I = (V_R/R) =$ ________ A

7. Peak voltage $V_R =$ ________ V

Peak current $I =$ ________ A

8. Original scope gain ________

Peak voltage $V_C =$ ________ V

10. Peak voltage $V_R =$ ________ V

$I =$ ________ A

11. $C =$ ________ μF

$V_R =$ ________ V

$I =$ ________ A

12. $C_1 =$ ________ μF

$C_2 =$ ________ μF

$V_R =$ ________ V

13. $V_R =$ ________ V

14. Frequency = ________ Hz

$V_R =$ ________ V

Time = ______ scale divisions

15. Frequency = ________ Hz

$V_R =$ ________ V

Time = ______ scale divisions

PART B. Phases

3. $\phi =$ ________ °

Questions

1.

2.

3.

4.

5.

EXPERIMENT B-2. Inductors

PART A. Inductive Reactances

1. $R =$ ________ Ω

4. Variable transformer setting = ________ V

   Peak-to-peak voltage V_R = ________ scale divisions

   = ________ V

5. Peak-to-peak voltage V_R = ________ V

6. Scope gain = ________

   Peak-to-peak voltage V_L = ________ V

8. Peak-to-peak voltage V_R = ________ V

   Peak-to-peak current $I = (V_R/R)$ = ________ A

9. $L =$ ________ H

   V_R = ________ V

10. Frequency = ________ Hz

    V_R = ________ V

    Period = ________ scale divisions

11. Frequency = ________ Hz

    V_R = ________ V

    Period = ________ scale divisions

PART B. Phase

2. $\phi =$ ________ °

EXPERIMENT B-3. Efficiency of a Transformer

2-6. Variable transformer setting = ________ V

Primary turns N_1 = ________

Secondary turns N_2 = ________

Load resistance $R_L(\Omega)$	open					
Primary voltage V_1 (V)						
Primary current I_1 (A)						
Primary phase angle ϕ						
Input power = $V_1 I_1 \cos\phi$ (W)						
Secondary voltage V_2 (V)						
Secondary current I_2 (A)						
Output power = $V_2 I_2$ (W)						
Efficiency = $P_{out}/P_{in} \times 100\%$						
Voltage ratio = V_2/V_1						
Current ratio = I_1/I_2						

Questions

1.

2.

3.

4.

5.

6.

7.

8.

9.

EXPERIMENT C-1. Resonance

1. $R =$ ________ Ω

   $L =$ ________ H

   $C =$ ________ μF

2. Resonant frequency = ________ Hz

3-4. V_{in} = ________ Hz

5.

Frequency (Hz)										
V_R (V)										

0-07-001727-1